MATH 131
Quiz VIII
April 19, 1991

Name *Key*

allowed 25 min
1 left at 11:00
3 started leaving at 16:00
next at 23:00
Many left at 2:20

$$\sum_{j=1}^{N} j = \frac{1}{2}N^2 + \frac{1}{2}N; \quad \sum_{j=1}^{N} j^2 = \frac{1}{3}N^3 + \frac{1}{2}N^2 + \frac{1}{6}N; \quad \sum_{j=1}^{N} j^3 = \frac{1}{4}N^4 + \frac{1}{2}N^3 + \frac{1}{4}N^2$$

3 1. Find

$$\sum_{j=2}^{4} (2j^2 - 3) = (2\cdot 2^2 - 3) + (2\cdot 3^2 - 3) + 2(4)^2 - 3$$

$$= 8 - 3 + 18 - 3 + 32 - 3 = 5 + 15 + 29 = \boxed{49}$$

17/27
7 close

3 2. Write in summation notation: 3 + 5 + 7 + 9 + 11 + 13

$$\sum_{j=1}^{6} (2j+1)$$

many forgot

20

15 3. Find each integral by any valid method:

a. $\int x^3 - \sqrt{x} + \cos x \, dx = \frac{x^4}{4} - \frac{x^{3/2}}{3/2} + \sin x + C$

$-x^{1/2}$

$$= \frac{x^4}{4} - \frac{2}{3}x^{3/2} + \sin x + C$$

notation, +C

12

13

b. $\int_0^1 \sqrt{5x + 4} \, dx$

$(5x+4)^{1/2}$

$$\left.\frac{(5x+4)^{3/2}}{3/2 \cdot 5}\right|_0^1 = \frac{2}{15}(9^{3/2}) - \frac{2}{15}4^{3/2}$$

$$= \frac{2}{15}(27 - 8) = \frac{38}{15}$$

or $u = 5x + 4$

$du = 5dx$

$$\int_{x=0}^{x=1} u^{1/2} \frac{1}{5} du = \left.\frac{u^{3/2}}{5 \cdot 3/2}\right|_{x=0}^{x=1}$$

$$= \left.\frac{2}{15}(5x+4)^{3/2}\right|_0^1 = \frac{38}{15}$$

c. $\int x^2 + 3\csc^2 x \, dx$

$$\frac{x^3}{3} - 3\cot x + C$$

10

9 4. Find the following integral using Riemann sums (check your answer):

8

$$\int_0^3 4x^3 \, dx$$

$\Delta x = \frac{3}{N}$

$c_j = j\left(\frac{3}{N}\right)$

$f(c_j) = 4\left(j\frac{3}{N}\right)^3$

$$\sum_{j=1}^{N} 4\left(j\cdot\frac{3}{N}\right)^3 \cdot \frac{3}{N} = \sum_{j=1}^{N} \frac{324}{N^4} j^3$$

$$= \frac{324}{N^4}\sum_{j=1}^{N} j^3$$

$$= \frac{324}{N^4}\left(\frac{1}{4}N^4 + \frac{1}{2}N^3 + \frac{1}{4}N^2\right)$$

$$\lim_{N\to\infty} \frac{324}{4}\frac{N^4}{N^4} = \frac{324}{4} = 81$$

(3) ck $\int_0^3 4x^3 dx = x^4\Big|_0^3 = 81$

5

2