MATH 131
Quiz IX
April 26, 1991

Name Key

allowed 28 min
First left @ 14
then 17 more 8 min
half done 27 min

10

1. Find the following integrals using any valid method:

a. $\int x \cos(x^2-2)\,dx$

$u = x^2-2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int \cos u \,\frac{1}{2}du$

$= \frac{1}{2}\sin u + C$

$= \frac{1}{2}\sin(x^2-2) + C$

7/28
8 close

b. $\int_{-1}^{2} \frac{x}{\sqrt{x+2}}\,dx$

$u = x+2$

$du = dx$

$x = u-2$

$x=-1 \quad u=1$

$x=2 \quad u=4$

$= \int_1^4 \frac{u-2}{\sqrt{u}}\,du = \int_1^4 u^{1/2} - 2u^{-1/2}\,du$

$= \frac{u^{3/2}}{3/2} - 2\frac{u^{1/2}}{1/2}\Big|_1^4$

$= \frac{2}{3}4^{3/2} - 4\cdot 4^{1/2} - \left(\frac{2}{3} - 2\right)$

$= \frac{2}{3}\cdot 8 - 8 - \frac{2}{3} + 2 = \frac{14}{3} - 6$

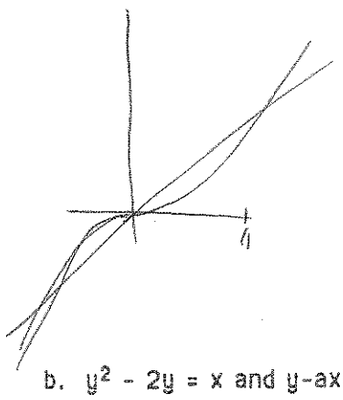
$= \boxed{\frac{2}{3}}$

7
4 close

12

2. Find the area of the regions bounded by the curves:

a. $y = x^3$ and $y = 4x$



$x^3 = 4x$

$x^3 - 4x = 0$

$x(x^2-4) = 0$

$x = 0, 2, -2$

$\int_{-2}^{0} x^3 - 4x\,dx + \int_0^2 4x - x^3\,dx$

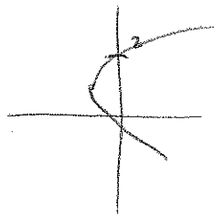
$= \frac{x^4}{4} - \frac{4x^2}{2}\Big|_{-2}^{0} + \frac{4x^2}{2} - \frac{x^4}{4}\Big|_0^2$

$= -\left(\frac{16}{4} - 2\cdot 4\right) + 8 - \frac{16}{4}$

$= 16 - 8 = 8$

(14)
1 close

b. $y^2 - 2y = x$ and y-axis.



$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0, 2$

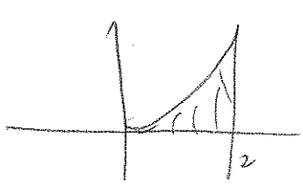
$\int_0^2 0 - (y^2 - 2y)\,dy = \int_0^2 -y^2 + 2y\,dy$

$= -\frac{y^3}{3} + y^2\Big|_0^2 = -\frac{8}{3} + 4 = \boxed{\frac{4}{3}}$

6

4

3. Find the volume of the solid generated by revolving about the x-axis the region bounded by the curves $y = x^2$, $x = 2$ and the x-axis.



$\int_0^2 \pi(x^2)^2\,dx = \int_0^2 \pi x^4\,dx$

$= \pi\frac{x^5}{5}\Big|_0^2 = \pi\frac{32}{5} - 0$

5

May simply didn't study!