MATH 131
Test I
February 22, 1991

Name/ Key Key

start

started to leave @ :35

$\frac{6}{30}$ left by :40

1/2 left @ :50

Lots checking

$\bar{X} = 79.5$

$m = 78.5$

1. Find the following derivatives: (42)

a. $f(x) = 3\sqrt{x} + 5x^7 + \dfrac{2}{x^4} - \cos x$, $f'(x) =$ $\frac{3}{2}x^{-1/2} + 35x^6 - 8x^{-5} + \sin x$

$= 3x^{1/2} + 5x^7 + 2x^{-4} - \cos x$

most missed this

most

b. $y = (x^3 - x^2 + 1)(2x - \sqrt{x} + 2)$, $\dfrac{dy}{dx} =$

$(x^3 - x^2 + 1)(2 - \frac{1}{2}x^{-1/2}) + (2x - \sqrt{x} + 2)(3x^2 - 2x)$

all

c. $f(x) = \dfrac{x^2}{\sin x}$, $f'(x) =$ $\dfrac{\sin x\,(2x) - x^2(\cos x)}{\sin^2 x}$

$\frac{23}{30}$

or $x^2(\sin x)^{-1}$

$2x(\sin x)^{-1} + x^2(-1)(\sin x)^{-2}\cos x$

d. $y = \sqrt{5x^2 + 2}$, $\dfrac{dy}{dx} =$ $\frac{1}{2}(5x^2+2)^{-1/2}(10x)$

$(5x^2+2)^{1/2}$

$= \dfrac{5x}{\sqrt{5x^2+2}}$

only one missed this

$\frac{26}{30}$

$\frac{16}{30}$

e. $y = \sin(3x^2 - 2)$, $y' =$ $+\cos(3x^2-2)(6x)$

$= +6x\cos(3x^2-2)$

most did as product

$\frac{16}{30}$

f. $f(x) = (x^2 + 4x - 3)^4$, $f''(x) =$

$f'(x) = 4(x^2+4x-3)^3(2x+4)$

$= (x^2+4x-3)^3(8x+16)$

$\frac{21}{30}$

$f''(x) = (x^2+4x-3)^3(8) + (8x+16)\,3(x^2+4x-3)^2(2x+4)$

$= 8(x^2+4x-3)^3 + 12(x^2+4x-3)^2(2x+4)^2$

42 $\frac{1}{3}$ got all

2. Complete the following DEFinition:
The function f is <u>continuous at the point x = a</u> if and only if (5)

(1) $f(a)$ exists

(2) $\lim_{x\to a} f(x)$ exist

(3) $\lim_{x\to a} f(x) = f(a)$

$\frac{9}{30}$

3. Find the following limits (± ∞ allowed), if they exist. (15)

a. $\lim_{x\to\infty} \frac{4x^3 - 2x + 1}{x + 2x^3} = \lim_{x\to\infty} \frac{4 - 2/x^2 + 1/x^3}{\frac{1}{x^3} + 2} = 2$

$\frac{4x^3}{2x^3} = 2$

all but 1

b. $\lim_{x\to 25} \frac{\sqrt{x} - 5}{x^2 - 25x} =$

$\frac{\sqrt{x}-5}{x^2 - 25x} \cdot \frac{\sqrt{x}+5}{\sqrt{x}+5} = \frac{x-25}{x(x-25)(\sqrt{x}+5)} = \frac{1}{x(\sqrt{x}+5)} \to \frac{1}{25(5+5)} = \frac{1}{250}$

$\frac{23}{30}$

c. $\lim_{x\to -3^+} \frac{4x}{x+3} = -\infty$

$\frac{-12}{+}$ $\quad -2.99 + 3 = +$

$\frac{22}{30}$

use another letter - confused with $x = a$

4. For what value of a will the function f be continuous? (4)

$$f(x) = \begin{cases} 0 & , x \le 0 \\ 3x & , 0 < x < 2 \\ a & , x \ge 2 \end{cases}$$

$\lim_{x\to 2} f(x) = \lim_{x\to 2} 3x = 6$

$a = 6$

must confused

$\frac{13}{30}$

5. Using only the DEFinition, find the derivative of: (Check your answer.) (10)
$f(x) = \frac{2}{3x - 1}$

more room

$$\lim_{\Delta x\to 0} \frac{\frac{2}{3(x+\Delta x)-1} - \frac{2}{3x-1}}{\Delta x} = \lim_{\Delta x\to 0} \frac{2}{\Delta x}\left(\frac{(3x-1) - (3(x+\Delta x)-1)}{(3(x+\Delta x)-1)(3x-1)}\right)$$

$$= \lim_{\Delta x\to 0} \frac{2}{\Delta x}\left(\frac{\cancel{3x} \cancel{-1} - \cancel{3x} - 3\Delta x \cancel{+1}}{(3(x+\Delta x)-1)(3x-1)}\right) = \lim_{\Delta x\to 0} \frac{2}{\cancel{\Delta x}} \frac{-3\cancel{\Delta x}}{(\ )(\ )}$$

$$= \lim_{\Delta x\to 0} \frac{-6}{(3(x+\Delta x)-1)(3x+1)} = \frac{-6}{(3x+1)^2}$$

stopped by algebra

$\frac{9}{30}$

2 close

$\frac{d}{dx} 2(3x-1)^{-1} = -2(3x-1)^{-2}(3)$

$= \frac{-6}{(3x-1)^2}$

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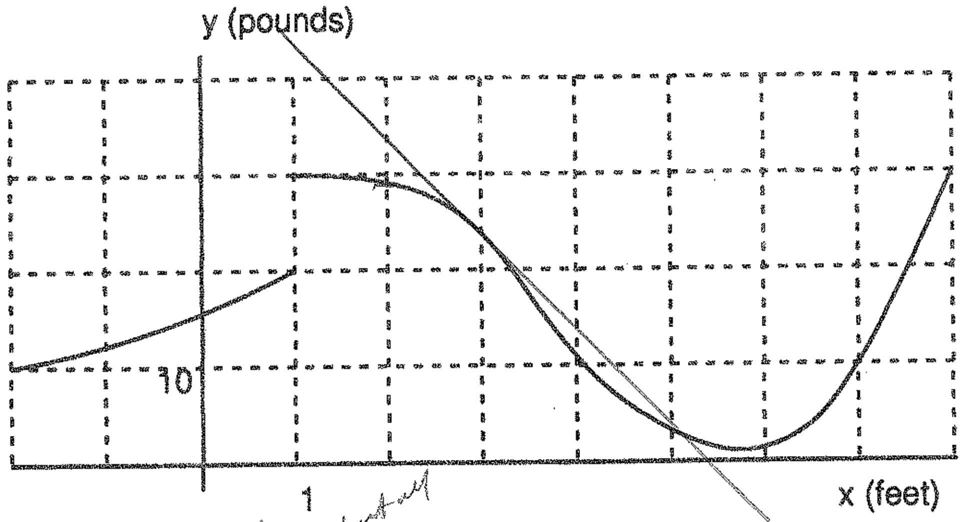
6. Suppose $y = f(x)$ is given by the following graph. (10)



a. $f(2) =$ 29 lb about

10/30

b. $\lim_{x \to 1^-} f(x) =$ 20 lbs most

~~most~~ close may

c. $f'(3) =$ −9.5 lb/ft   (3, 23)   (5, 4)

$\frac{23-4}{\cancel{3-5}} = -\frac{19}{2} = -9.5$

7. Find the equation of the line tangent to the curve $y = 12\sqrt{x} - x^2$ at $x = 4$. (8)

$\frac{dy}{dx} = 12 \frac{1}{2} x^{-1/2} - 2x$

$x = 4$

$y = 12\sqrt{4} - 4^2$

$= 24 - 16 = 8$

16/30

$x = 4 \quad m = \frac{dy}{dx} = 6\frac{1}{\sqrt{4}} - 8$

$= \frac{6}{2} - 8 = 3 - 8 = -5$

$y - 8 = -5(x - 4)$

$y = -5x + 20 + 8$

$\boxed{y = -5x + 28 \quad \cancel{y = -x + 12}}$

8. For what values of x is the function f not continuous? (6)

$$f(x) = \begin{cases} 3x, & x < -1 \\ \frac{6}{x}, & -1 \le x \le 2 \\ x + 1, & x > 2 \end{cases}$$

$x = -1 \quad x = 0, \quad x = 2$

$x = 0$ no not defined

12/30

$x = 2 \quad \lim_{x \to 2^+} = 3$ ✓

$\lim_{x \to 2^-} = 3$

11 wrong 0

$x = -1 \quad \lim_{x \to -1^-} 3x = -3$ X

cut $\lim_{x \to -1^+} \frac{6}{-1} = -6$

0, −1

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