Maybe long mean 64.6
mod = 70

MATH 131
Test II
April 5, 1991

Name Key

1. Find all the critical points of $f(x) = 3x^4 - 8x^3 + 6x^2 + 3$. (10)

$f'(x) = 12x^3 - 24x^2 + 12x = 0$

$12x(x^2 - 2x + 1) = 0$

$12x(x-1)^2 = 0$

$x = 0, 1$

20 OK
6 alg
1 x

2. Find the (absolute) maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 1$ for $-1 \le x \le 2$. Justify. (10)

$f'(x) = 3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0, 4$

$(-1) - 6 + 1 \qquad 8 - 24 + 1 = -15$

	x	y	
EP	-1	-6	
CP	0	1	MAX
EP	2	-15	MIN

13
7 Chain

3. The surface area of a sphere is given by $A = 4\pi r^2$. If the radius of a balloon is increasing at the rate of 2 in. per second when the radius is 3 in., how fast is the surface area changing? (10)

$\frac{dr}{dt} = 2$ when $r = 3$ $\qquad \frac{dA}{dt} = ?$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(3)(2) = 48\pi$ sq in/sec

150.8

14 OK on chain

4. Find the derivative dy/dx when $x^3y + xy^2 + y^2 = 4$. (10)

$3x^2y + x^3\frac{dy}{dx} + x2y\frac{dy}{dx} + y^2 + 2y\frac{dy}{dx} = 0$

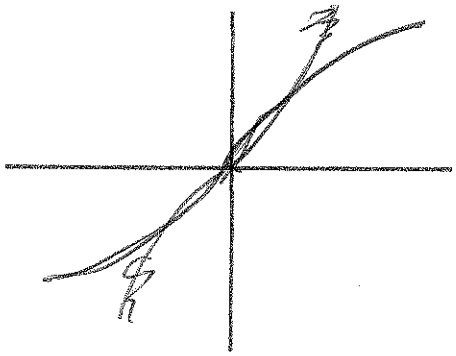
$(x^3 + 2xy + 2y)\frac{dy}{dx} = -y^2 - 3x^2y$

$\frac{dy}{dx} = \frac{-y^2 - 3x^2y}{x^3 + 2xy + 2y}$

14
6 chain

5. FREE (5)

6. Draw the graph of a function for which $f(0) = 0$, $f'(x) > 0$ for all x, and $f''(x) < 0$ for $x > 0$ and $f''(x) > 0$ for $x < 0$. (5)



6
$19 - \frac{1}{2}$
4 nothing

7. Find the equation of the straight line which is tangent to the curve $x \sin y = 2$ at the point $(2, \pi/2)$. (10)

$x \cos y \frac{dy}{dx} + \sin y = 0$

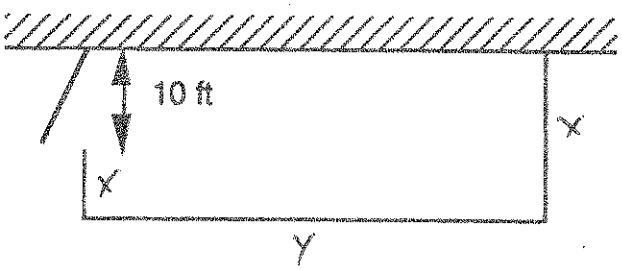
$\frac{dy}{dx} = \frac{-\sin y}{x \cos y}$

$x = 2$ (one & done)

$x = 2$
$y = \frac{\pi}{2}$ $\quad \frac{dy}{dx} = \frac{-1}{2(0)}$ (oops ∞)

not a problem for most → all but 7 more way off (no implicit diff)

8. A rectangular pen is to be built so that one side is against a building (so no fencing is needed there) and one side will have a 10 foot gate. There is 150 feet of fencing available. What are the dimensions of the largest pen that can be built? (10)



4

$A = (x+10)y$

$x + x + 10 + y = 150$ (adj)

$y = 140 - 2x$

$A = (x+10)(140 - 2x)$

$A = xy$

$y + x + (x-10) = 150$

$y + 2x - 10 = 150$

$y = 160 - 2x$

$2x = 160 - y$

$x = 80 - y/2$

$A = (80 - y/2)y$

$A = x(160 - 2x) \quad 10 \le x \le 80$

$= 160x - 2x^2$

$\frac{dA}{dx} = 160 - 4x = 0$

$x = 40$ 2

x	y
10	10(140) = 1400
40	40(80) = 3200
80	0

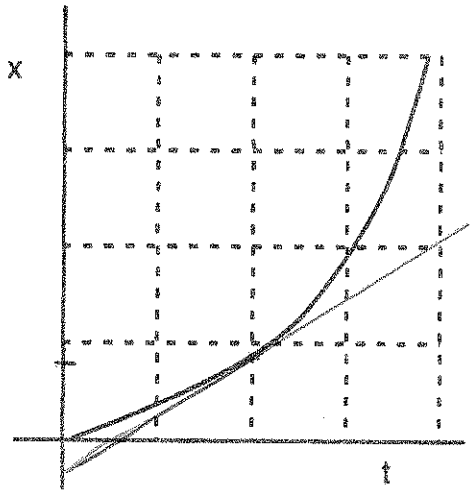
40 × 80

30

9. $y = x^3$, and $x = f(t)$ is given by the graph below. When $t = 2$ find:

a. y.

b. dy/dt.



a. $t=2$ $x=.8$ $y=(.8)^3$ (10)

$=$ .512

b. $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$ ↑ $t=2$ ↑ $x=.8$

I got all right must get back.

$\frac{dy}{dx} = 3x^2 = 3(.8)^2 = 1.92$

$t=2$ (2, .5) (4, 2)

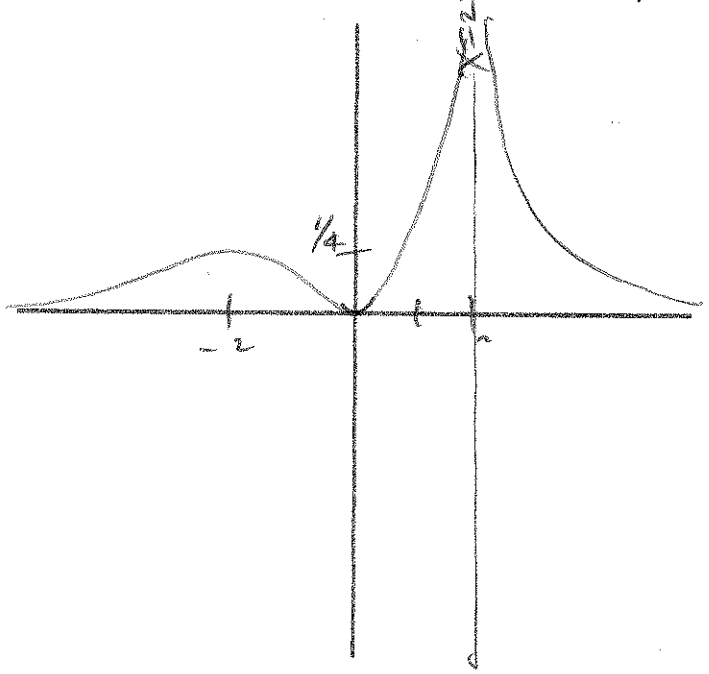
$\frac{dx}{dt} = \frac{2-.5}{4-2} = .75$

$\frac{dy}{dt} = (1.92)(.75) =$ 1.44 1.152

10. Sketch the graph of (20)

$$f(x) = \frac{16x^2}{(x-2)^4}$$

Give x-y coordinates of critical points and equations of asymptotes (if any).



(3)

zeros $x=0$

VA $x=2$

$\frac{16x^2}{x^4} = \frac{16}{x^2} \to 0$

HA $y=0$

$$f'(x) = \frac{(x-2)^4 32x - 16x^2 4(x-2)^3}{(x-2)^8}$$

$$= \frac{(x-2)^3 32x[4(x-2) - 2x]}{(x-2)^8}$$

$$= \frac{32x[-2-2x]}{(x-2)^5}$$

$x=0$, $x=-2$

(0, 0)

$\frac{16(4)}{4^4} = \frac{1}{4}$

$(-2, \frac{1}{4})$

30