Time OK

Fust left at 1:18
Some at 1:30 (checking)
more at 1:40 (8)
mod time 1:51
16 left by 2:00
Lorem
last at 2:10

$\frac{112}{175}$

MATH 131
Final Exam
May 4, 1993 1 – 10 pt problem FREE (except those who want it)

Name Key mod 4

S-U? ___
(~~2~~ 110)

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. Find the derivative of: (20)

a. $x^4 \sin x + \csc 4x$

$x^4 \cos x + 4x^3 \sin x - \csc 4x \cot 4x \cdot 4$

many forgot prod rule

16

too hard

b. $\dfrac{\tan x}{\sqrt{x} + x}$

$$\frac{(\sqrt{x}+x)\sec^2 x - \tan x\left(\frac{1}{2\sqrt{x}}+1\right)}{(\sqrt{x}+x)^2}$$

V8 close

some tried
$\tan x\,(x^{-1/2}+x^{-1})$

c. $\ln \sqrt{x^2 - 3}$

or $\frac{1}{2}\ln(x^2-3)$

too hard

$$\frac{1}{\sqrt{x^2-3}}\,\frac{1}{2}(x^2-3)^{-1/2}(2x) = \frac{x}{x^2-3}$$

$\frac{1}{2}\,\frac{1}{x^2-3}\cdot 2x$

3

2. Find: (7)

$$\lim_{x \to -5} \frac{x^2 - 25}{x + 5} = \lim_{x\to -5} \frac{(x-5)\cancel{(x+5)}}{\cancel{x+5}} = -10$$

5

all but 4

3. The acceleration on the moon due to gravity is -5 ft/sec². If an object is shot upward with an initial velocity of 100 ft/sec., how high does it go? (10)

5

$a = -5$
$v = -5t + c$
$100 = 0 + c$
$v = -5t + 100$
$s = -\frac{5}{2}t^2 + 100t + c$
$0 = 0 + c$
$s = -\frac{5}{2}t^2 + 100t$

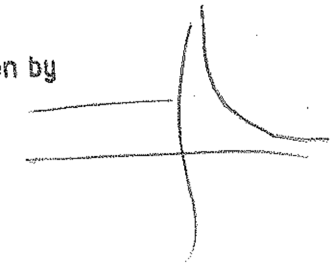
$v = 0$
$-5t + 100 = 0$
$t = 20$

$s = -\frac{5}{2}(20)^2 + 100(20)$
$= -1000 + 2000$
$= 1000$ ft

37

4. Let the function f be given by (15)

$$f(x) = \begin{cases} 3, & x \le 0 \\ \dfrac{x+1}{x^2}, & x > 0 \end{cases}$$



good Q

$g(x) = x^2$

a. $\lim_{x \to 0^-} f(x) =$ 3

12 got all

b. $\lim_{x \to 0^+} f(x) =$ ∞

c. $\lim_{x \to \infty} f(x) =$ 0

d. $\lim_{x \to -\infty} f(x) =$ 3

e. $f(g(2)) =$ $f(4) = \frac{5}{16}$

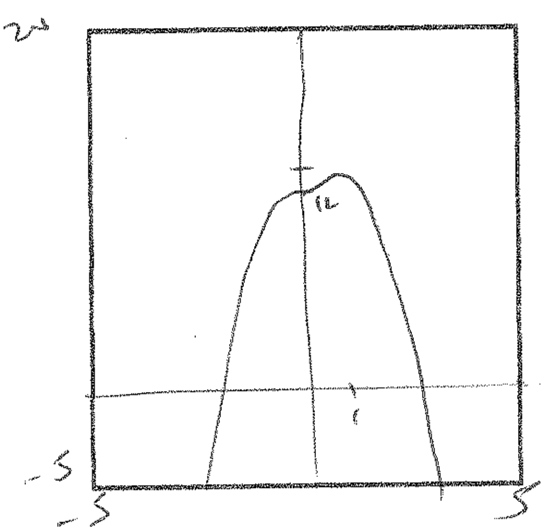
5. Sketch the graph of $f(x) = 4x^3 - 3x^4 + 12$, and find (analytically) (15)

a. coordinates (if any) of critical points

b. coordinates (if any) of points of inflection

c. intervals(s) for which the graph is increasing, decreasing

d. intervals(s) for which the graph is concave up and concave down.



$f'(x) = 12x^2 - 12x^3 = 0$ 16

$12x^2(1-x) = 0$

$x = 0, 1$

$(0, 12)$ $(1, 13)$

$f''(x) = 24x - 36x^2$

$12x(2-3x)$

$x = 0, \frac{2}{3}$

$(0, 12)$ $(\frac{2}{3}, 12.59)$

I $(-\infty, 1)$

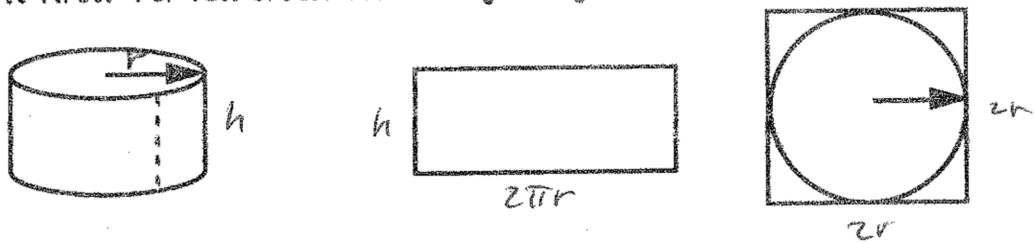
D $(1, \infty)$

CC↑ ~~$(-\infty, 0)$~~ $(0, \frac{2}{3})$

CC↓ $(-\infty, 0)$ $(\frac{2}{3}, \infty)$

30

6. A cylindrical box with open top is to hold 4 ft^3. The circular bottom will be cut from a square piece, the rest will be waste, and the side is a rectangular piece rolled up. What dimensions (radius of the base and height) will use the least amount of <u>total</u> material (including waste)? [Find the function and sketch it first. For full credit solve analytically. Hint: Vol = $\pi r^2 h$] (10)

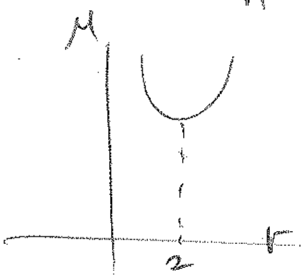


~~14 free~~
more (most piece)

$\pi r^2 h = 4 \qquad M = 2\pi r h + (2r)^2$

$h = \frac{4}{\pi} r^{-2} \to M = 2\pi r\left(\frac{4}{\pi r^2}\right) + 4r^2$

$= 8r^{-1} + 4r^2$



$\frac{dM}{dr} = -8r^{-2} + 8r = 0$

$-\frac{8}{r^2} + 8r = 0$

$\frac{-8+8r^3}{r^2} = 0 \qquad r \neq 2) \qquad h = \left(\frac{1}{\pi}\right)$

7. Find the equation of the straight line which is tangent to the curve $x^3 - xy + y^2 = 4$ at the point $(-1,2)$. (10)

$3x^2 - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$

$3x^2 - y + (2y - x)\frac{dy}{dx} = 0$

$\underbrace{3(-1)^2 - 2}_{1} + (4+1)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{1}{5}$

$y - 2 = -\frac{1}{5}(x+1)$

$y - 2 = -\frac{1}{5}x - \frac{1}{5}$

$y = -\frac{1}{5}x + \frac{9}{5}$

8. Find the following integral <u>using the definition.</u> (Find the limit of Riemann sums for n subintervals using the right end point in each interval: RR_n.) (10)

$\int_0^4 3x^2\,dx \qquad \Delta x = \frac{4}{n} \qquad 0 \quad \frac{4}{n} \quad 2\frac{4}{n} \quad \cdots \quad n\cdot\frac{4}{n}$

10

$c_k = k\frac{4}{n}$

$\sum_{k=1}^{n} 3\left(k\frac{4}{n}\right)^2 \frac{4}{n} = \frac{192}{n^3}\sum_{k=1}^{n} k^2 = \frac{192}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$= \frac{192}{6}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$

$= 32\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) \to 64$

$\frac{64}{3}$
112

30

9. A bacteria count is increasing at a rate proportional to the current count. There was a count of 5000 at the beginning, and 3 hours later there was a count of 12,000. What will be the count after 5 hours? (8) 10

$y = 5000e^{kt}$

$12000 = 5000e^{k3}$

$\ln \frac{12}{5} = k3$

$k = \frac{1}{3}\ln\left(\frac{12}{5}\right)$

$y = 5000\, e^{\frac{1}{3}\ln\left(\frac{12}{5}\right)t}$

$t = 5$

$y = 5000\, e^{\frac{1}{3}\ln\left(\frac{12}{5}\right)\cdot 5}$

$= 21511$

16

10. Find the following integrals: (20)

a. $\int 4x^2 - \frac{3}{\sqrt{x}} + \sin x \, dx$ $= \int 4x^2 - 3x^{-1/2} + \sin x\, dx$

$= \frac{4x^3}{3} - 3\frac{x^{1/2}}{1/2} - \cos x + c$

$= \frac{4}{3}x^3 - 6\sqrt{x} - \cos x + C$

16

b. $\int \frac{x}{\sqrt{x^2 - 4}} dx$ $= \frac{1}{2}\int u^{-1/2} du = \frac{1}{2}\frac{u^{1/2}}{1/2} + c$

$= \sqrt{x^2-4} + c$

$u = x^2 - 4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

10

many got u

c. $\int_0^2 x\sqrt{x+1}\, dx$

NONE

I close

$u = x+1$ $x = u-1$ $x=0$ $u=1$

$x=2$ $u=3$

$du = dx$

$\int_1^3 (u-1)\sqrt{u}\, du = \int_1^3 u^{3/2} - u^{1/2}\, du = \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\Big|_1^3$

$= \frac{2}{5}3^{5/2} - \frac{2}{3}3^{3/2} - \left(\frac{2}{5} - \frac{2}{3}\right)$

$= \frac{18}{5}\sqrt{3} - 2\sqrt{3} + \frac{4}{5}$

$= \frac{8}{5}\sqrt{3} + \frac{4}{5}$

28

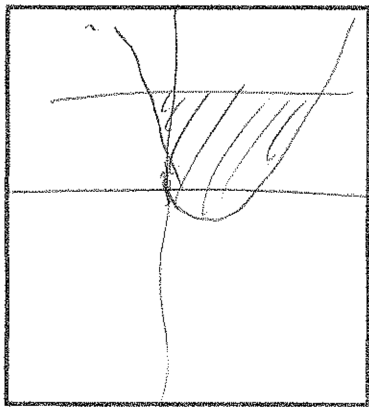
11. Express this limit of a Riemann sum as a definite intgeral: (5)

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{c_k^2 - 1}\,\Delta x, \text{ on } [2\ \ 4].$$

$$\int_2^4 \frac{3}{x^2-1}\,dx$$

18

12. Find the area between the curves $y = x^2 - 2x$ and $y = 3$. Sketch the region.(10)



$$\int_{-1}^{3} 3-(x^2-2x)\,dx$$

19

$$= \int_{-1}^{3} -x^2+2x+3\,dx$$

$$= -\frac{x^3}{3} + \frac{2x^2}{2} + 3x \Big|_{-1}^{3}$$

$$= -\frac{3^3}{3} + 3^2 + 3^2 - \left(\frac{1}{3} + 1 - 3\right)$$

$$= \cancel{-9} + \cancel{9} + 9 - \frac{1}{3} + 2 = 11 - \frac{1}{3} = 10\frac{2}{3}$$

13. The amount y of a radioactive substance (grams) after t hours is given by

$$y = 500\, e^{-\frac{t \ln 2}{267}}$$

a. What amount was there initially?

b. What is the half-life (time until one half the amount is left)?

most missed → c. How fast is the amount changing after 100 hours?

d. When will 10% be left? (15)

a. $t=0$ $\boxed{500}$

b. $\frac{1}{2} = e^{-\frac{t\ln 2}{267}}$

$\ln\frac{1}{2} = -\frac{t\ln 2}{267}$

$t = 267$

seems too hard

c. $\frac{dy}{dt} = 500\, e^{-\frac{t\ln 2}{267}}\left(-\frac{\ln 2}{267}\right)$

$= 500\, e^{-\frac{100\ln 2}{267}}\left(-\frac{\ln 2}{267}\right)$

$= -1.00124$

d. $.10 = e^{-\frac{t\ln 2}{267}}$

$\ln(.10) = -\frac{t\ln 2}{267}$

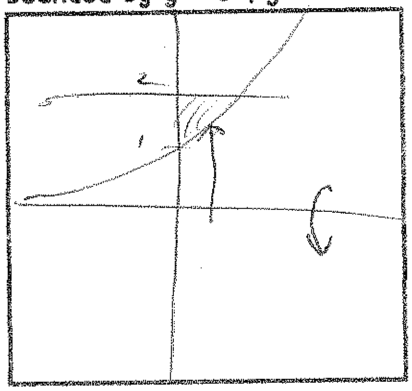
$t = \frac{267\ln(.1)}{\ln 2}$

$t = 886.9$

18 all or close

$\frac{}{30}$

14. Find the volume of the solid generated by revolving about the x-axis the region bounded by $y = e^x$, $y = 2$ and the y-axis. Sketch the region first. (10)



$$\int_0^{\ln 2} 2^2\pi - \pi(e^x)^2\,dx$$

$$= \pi \int_0^{\ln 2} 4 - e^{2x}\,dx$$

$$= \pi\left(4x - \tfrac{1}{2}e^{2x}\right)\Big|_0^{\ln 2}$$

$$= \pi\left(4\ln 2 - \tfrac{1}{2}e^{2\ln 2} - \left(0 - \tfrac{1}{2}\right)\right)$$

$$= \pi\left(4\ln 2 - \tfrac{1}{2}4 + \tfrac{1}{2}\right)$$

$$= \pi(4\ln 2 - 1.5)$$

$$= 1.27\pi$$

$$\approx 3.9979 \approx 4$$

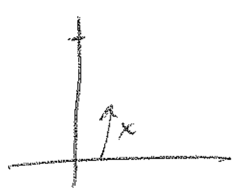
4 all
6 close

7 did
$\int \pi (R-r)^2$

15. A 20 ton anchor is at the end of a 90 foot chain. The chain weighs 100 pounds per foot. How much work is done in lifting the anchor and chain to the winch at the top? (10)

chain anchor $20(2000)(90) = 3{,}600{,}000$ ft lb

7



$(90-x)\cdot 100$

$$100\int_0^{90}(90-x)\,dx = 100\left(90x - \frac{x^2}{2}\right)\Big|_0^{90}$$

$$= 100\left(90^2 - \tfrac{1}{2}90^2\right)$$

$$= 100\left(\frac{90^2}{2}\right) = 50\cdot 90^2 = 405{,}000$$

$81 \cdot 5 = 405$

$3{,}600{,}000 + 405{,}000$

$= 4{,}005{,}000$ ft-lbs

20