Time OK, not long. Time to check

MATH 131
Test I
February 19, 1993

Name KEY

First left @ 29
checking grading at :40
Next left @ :40
(35) more
Bunch at 48
10 by 44

1. Find the derivatives of each of the following:

a. $f(x) = 3\sqrt{x} - 3x^6 + \frac{2}{x^4} - \tan x,\quad f'(x) = \boxed{\frac{3}{2}x^{-1/2} - 18x^5 - 8x^{-5} - \sec^2 x}$

$3x^{1/2} - 3x^6 + 2x^{-4} - \tan x$

20 all

b. $y = \frac{x^3 + 4}{\tan x}$ $\quad \frac{dy}{dx} = \frac{\tan x\,(3x^2) - (x^3+4)\sec^2 x}{\tan^2 x}$

good choice
prod - chain rule is harder

18
some way off

9 all

c. $y = \frac{4}{\sqrt{3x^2 - 2}} = 4(3x^2-2)^{-1/2}$

$\frac{dy}{dx} = -\frac{4}{2}(3x^2-2)^{-3/2}(6x) = \frac{-12x}{(3x^2-2)^{3/2}}$

most made this too hard. $\pm\frac{1}{2}$ next time

d. $y = \sec(4x)$

$\frac{dy}{dx} = \sec 4x \tan 4x \cdot 4$

17
many did product rule

e. $y = \tan(\cos x)$

$\frac{dy}{dx} = \sec^2(\cos x)(-\sin x)$

14
many product rules

f. $y = (2x - 3)^4$, $\frac{d^2y}{dx^2} = 24(2x-3)^2 \cdot 2$

earlier

$= 48(2x-3)^3$

$\frac{dy}{dx} = 4(2x-3)^3 \cdot 2$

$= 8(2x-3)^3$

12 all

g. $y = \sqrt{x^2 + \sin(4x - 1)}$

$= (x^2 + \sin(4x-1))^{1/2}$

10 all

$\frac{dy}{dx} = \frac{1}{2}(x^2 + \sin(4x-1))^{-1/2}(2x + \cos(4x-1)\cdot 4)$

$\overline{35}$

much due Fri 3/1

2. Find the following limits ($\pm\infty$ allowed), if they exist. (20)

fewer?

a. $\lim\limits_{x \to \infty} \dfrac{-3x^2 - 2x + 1}{x + 4}$ $= \lim\limits_{x\to\infty} \dfrac{-3x - 2 + \frac{1}{x}}{1 + \frac{4}{x}} \to -\infty$ 17

b. $\lim\limits_{x \to 5} \dfrac{x - 5}{x^2 - 25}$ $\lim\limits_{x\to 5} \dfrac{\cancel{x-5}}{\cancel{(x-5)}(x+5)} = \lim\limits_{x\to 5} \dfrac{1}{x+5} = \dfrac{1}{5+5} = \dfrac{1}{10}$ 20

c. $\lim\limits_{x \to 3} \dfrac{4x + 3}{x + 3}$ $= \dfrac{4\cdot 3 + 3}{6} = \dfrac{15}{6} = \dfrac{5}{2}$ 24

d. $\lim\limits_{x \to -3^+} \dfrac{2x + 3}{x + 3}$ $-\infty$ 16

$\dfrac{-6+3}{+}$

3. For the function (10)

$$f(x) = \begin{cases} 0 & , x \le 0 \\ 3x & , 0 < x < 2 \\ K & , x \ge 2 \end{cases}$$

a. What value of K will make the function f continuous at x = 2 (if any)?

$K = \lim\limits_{x\to 2} 6x = 6$ 6 all

b. What is the limit at x = 0 (is any)? 0

c. What is f(g(2)) where g(x) = 3x - 5

$f(g(2)) = f(1) = \boxed{3}$

4. Find f'(x) using only the definition. (10)

$f(x) = \sqrt{2x + 1}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{} + \sqrt{}}$$

$2x + 2h + 1 - 2x - 1$

$$= \frac{2(x+h)+1 - (2x+1)}{h\left[\sqrt{2(x+h)+1} + \sqrt{2x+1}\right]} =$$

12 all

$$= \frac{2h}{h\left[\sqrt{} + \sqrt{}\right]} = \frac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

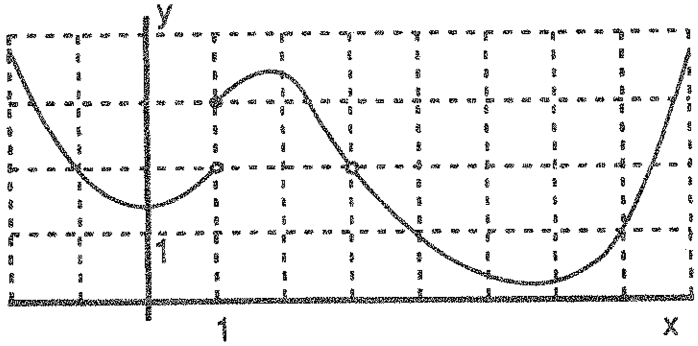
$$\longrightarrow \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}} = \frac{2}{2\sqrt{2x+1}}$$

$$= \frac{1}{\sqrt{2x+1}}$$

40

5. Suppose $y = f(x)$ is given by the following graph. (15)

~~feature~~?



2 a. $f(1) =$ ~~2~~ 3

3 b. $f(f(-1)) = f(2) =$ ~~3~~ 4

3 c. $\lim_{x \to 1^-} f(x) = 2$

3 d. $\lim_{x \to 3} f(x) = 2$

2 e. Is $f'(4) > 0$? or < 0? <0 ← lots missed

3 f. For what values of x is the function f not continuous?

1, 3

3 all
16 close

6. The height (ft.) of an object above the ground is $s = 200 + 128\,t - 16\,t^2$, where t is time in seconds. (Include units with answers.) (10)

v-3

2 a. What is the initial velocity? up or down? (confused initial)

2 b. What is the velocity after 3 seconds? Up or down?

3 c. How high does the object go? Some used graph

5 all
15 close
(a or b)
usually

a. $v = 128 - 32t$

$t = 0 \quad v = \boxed{128 \text{ ft/sec up}}$

b. $t = 3 \quad v = 128 - 96 = \boxed{32 \text{ ft sec up}}$

c. $v = 0 \quad t = \frac{128}{32} = 4$

$S = 200 + 128(4) - 16(16)$

$= 200 + 512 - 256$

$= 200 + 256 = \boxed{456 \text{ ft}}$

25