Time OK
Med 68
$\bar{x}$ 65.1

MATH 131
Test II
April 2, 1993

Name Key

1. Find the following derivative: (8)

$\frac{dy}{dx}$ where $3x^3 - x^2y + \cos y = 3$..

some
some

$$9x^2 - \left(x^2\frac{dy}{dx} + y2x\right) - \sin y\,\frac{dy}{dx} = 0$$

15

$$\left(-x^2 - \sin y\right)\frac{dy}{dx} = -9x^2 + 2xy$$

$$\frac{dy}{dx} = \frac{2xy - 9x^2}{-\sin y - x^2}$$

2. Solve for y as a function of x: (15)

a. $\frac{dy}{dx} = x + 2\sin(3x)$.

9 11

$$y = \frac{x^2}{2} - 2\frac{\cos 3x}{3} + c$$

b. $\frac{dy}{dx} = 4x^3 - \frac{3}{x^4}$, $y = 5$ when $x = 1$.

11

$$= 4x^3 - 3x^{-4}$$

$$y = x^4 - \frac{3x^{-3}}{-3} + c$$

$$y = x^4 + x^{-3} + c$$

$$5 = 1 + 1 + c$$
$$c = 3$$

$$y = x^4 + \frac{1}{x^3} + 3$$

3. In the metric system the acceleration due to gravity is a = - 9.8 m/sec/sec. If at t = 0 sec, the velocity is v = 12 m/sec up and position is s = 100 m above ground. What is the position when t = 2 sec? (10)

5
Some close

$a = -9.8 \quad t = 0 \quad v = 12 \quad s = 100$

$v = -9.8t + c \quad 12 = 0 + c$

$v = -9.8t + 12$

$s = -4.9t^2 + 12t + C$

$100 = 0 + c$

$s = -4.9t^2 + 12t + 100$

$t = 2$

$s = -4.9(4) + 12(2) + 100$

$= 104.4$ m

many completely missed (thought it was harder)

4. A circle has a radius of approximately 15 in. We know the radius within ±.1 in. What is the approximate error in the area? (Use diff) (8)

$A = \pi r^2$

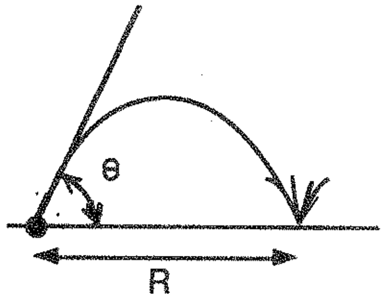
$dA = 2\pi r\, dr \qquad dr = \pm .1$

$dA = 2\pi(15)(.1) = \pm 3\pi$ sq in

15

5. The range R (in feet) of a sweeping-type lawn sprinkler is given by the formula $R = 100 \sin(2\theta)$, $0 \le \theta \le \pi$. (R = distance to where the water hits) where θ is the angle above the horizontal. If the angle is changing at the rate of 0.1 radian/sec when the angle is $\pi/3$, how fast is the spot where the water hits moving along the ground? (10)

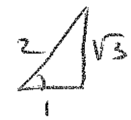
14



$R = 100 \sin(2\theta) \quad 0 \le \theta \le \pi$

$\frac{dR}{dt} = 100 \cos(2\theta) \cdot 2 \frac{d\theta}{dt}$

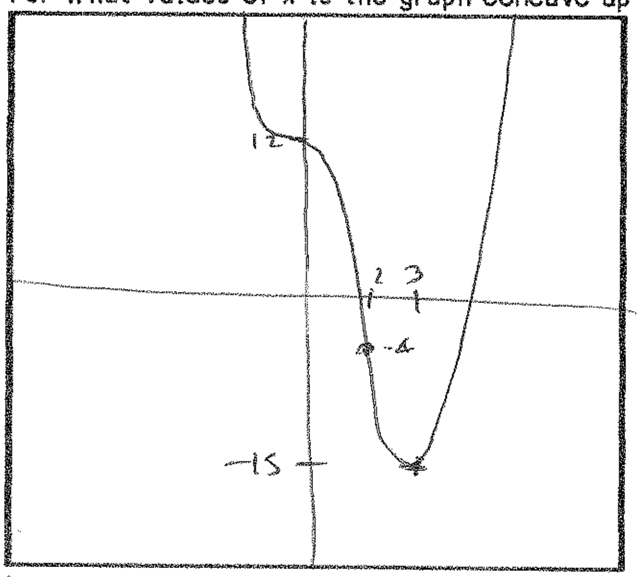
$\frac{d\theta}{dt} = .1 \quad \theta = \frac{\pi}{3}$



$\frac{dR}{dt} = 200 \cos\left(\frac{2\pi}{3}\right) \cdot (.1)$

$= 20\left(-\frac{1}{2}\right) = -10$ ft/sec

6. Sketch the graph of $f(x) = x^4 - 4x^3 + 12$ in the box. Give range used. Analytically find x-y coordinates of critical points and points of inflection. For what values of x is the function increasing? decreasing? For what values of x is the graph concave up? concave down? (15)



$f(x) = x^4 - 4x^3 + 12$

$f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x-3)$

$x = 0, 3$ CP

$f''(x) = 12x^2 - 24x = 0$

$12x(x-2)$

$x = 0, 2$

$f(0) = 12 \quad f(3) = -15$

$f(2) = -4$

I $(3, \infty)$

D $(-\infty, 3)$

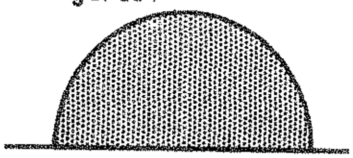
cc↑ $(-\infty, 0)$ $(2, \infty)$

cc↓ $(0, 2)$

10 all
many close
few way off

25

7. Oil from a pipeline on shore is leaking into the ocean at a constant rate. A semicircular oil slick is forming, and its area is increasing at the rate of 20 sq. yd. per hour. How fast is the radius changing when the slick has a radius of 100 yards? (10)



$A = \frac{1}{2}\pi r^2$

$\frac{dA}{dt} = \frac{1}{2} 2\pi r \frac{dr}{dt} = \pi r \frac{dr}{dt}$

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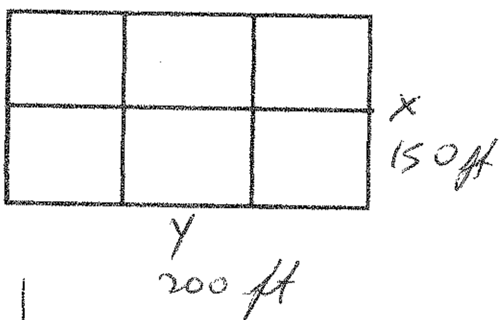
$\frac{dA}{dt} = 20 \quad r = 100$

$20 = \pi(100)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{20}{100\pi} = \frac{.2}{5\pi} = .064\ \text{yd}/\text{hr}$

8. A corral is to be made with 6 compartments as shown. If the amount of fence available is 1200 ft, what dimensions will yield the largest corral? (10)



$3y + 4x = 1200 \qquad y = \frac{1200 - 4x}{3} = 400 - \frac{4}{3}x$

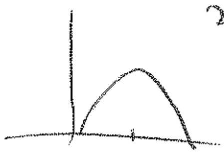
$A = xy$

12

many got backwards

$A = x(400 - \frac{4}{3}x)$

$= 400x - \frac{4}{3}x^2 \qquad 0 \le x \le 300$



$\frac{dA}{dx} = 400 - \frac{8}{3}x = 0$

$\frac{8}{3}x = 400$

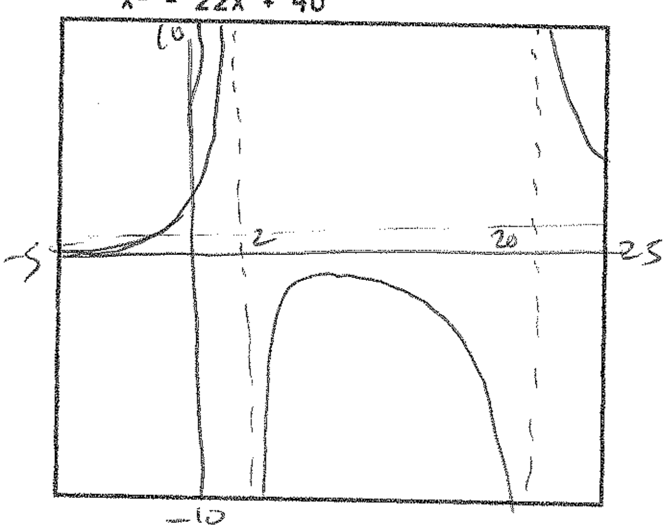
$x = \frac{400 \cdot 3}{8} = 150 \qquad y = 400 - 200 = 200$

200 × 150 ft

9. Sketch the graph of the following. Give equations of asymptotes (if any). Give range. (14)

complete draw fewer

$y = \dfrac{x^2 + 25}{x^2 - 22x + 40}$



$\lim_{x\to\infty} \frac{x^2+25}{x^2-22x+40} = \lim_{x\to\infty} \frac{1+\frac{25}{x^2}}{1-\frac{22}{x}+\frac{40}{x^2}} = 1$

$\boxed{y = 1 \quad HA}$

$x^2 - 22x + 40$

$(x-20)(x-2) = 0 \qquad x = 2,\ x = 20$ VA

2

HA 4
VA 4
G 4

34

draw asympt.