Math 131
Final Exam
March 19, 1970

Name *Key* 12

Show work! Each problem worth 10 points.

1. Given $\epsilon > 0$ find $\delta > 0$ which satisfies the definition of the limit

$$\lim_{x \to 2} (5-3x) = -1 .$$

$$|5-3x-(-1)| < \epsilon$$
$$|6-3x| < \epsilon$$
$$3|2-x| < \epsilon$$
$$|x-2| < \epsilon/3 \qquad \delta = \epsilon/3$$

2. $$g(x) = \begin{cases} x^3-1, & x \neq 3 \\ 28, & x = 3 \end{cases} \qquad h(x) = \begin{cases} x^3, & x \geq 3 \\ -x^3, & x < 3 \end{cases}$$

a) $\lim_{x \to 3} g(x) =$ 26 $\quad 27-1=26$

b) Is h a continuous function? YES( ) NO(✓)

3. a) $\lim_{x \to 2} \frac{x^3-x+1}{x+3} = \frac{2^3-2+1}{2+3} = \frac{8-1}{5} = \frac{7}{5} = 1\frac{2}{5}$

b) $\lim_{x \to -4} \frac{x^2-16}{x+4} = \lim_{x \to -4} \frac{(x-4)\cancel{(x+4)}}{\cancel{x+4}} = -8$

4. Find $\lim_{x \downarrow 0} (\sqrt{x} - 1)$ and $\lim_{x \uparrow 0} (\sqrt{x} - 1)$.

$-1 \qquad$ DNE

Differentiate the following (5-8):

5. $3x^{\overset{3}{2}} + x^2 - x + \frac{10}{x^3}$

$10x^{-3}$ $\quad 9x^2 + 2x - 1 - 30x^{-4}$

$9x^2 + 2x - 1 - \frac{30}{x^4}$

6. $(5x^2 + 4)^8$

$8(5x^2+4)^7 \, 10x = 80x(5x^2+4)^7$

7. $(x+2)^8(x-3)^{12}$

$8(x+2)^7(x-3)^{12} + (x+2)^8 \, 12(x-3)^{11}$

$4(x+2)^7(x-3)^{11}\left(\underset{-6}{2x-3} + \underset{+6}{3(x+2)}\right)$

$4(x+2)^7(x-3)^{11}(5x) = 20x(x+2)^7(x-3)^{11}$

8. $\frac{x}{x^3 - 4}$

$\frac{(x^3-4) - x(3x^2)}{(x^3-4)^2} = \frac{-2x^3-4}{(x^3-4)^2}$

9. Find the equation of the straight line which is tangent to the curve xy = 1 at the point ($\frac{1}{2}$,2).

$y = \frac{1}{x} = x^{-1}$ $\quad$ 6 $\quad y = -4x + b$

$f'(x) = -x^{-2} = -\frac{1}{x^2}$ $\quad 2 = -4(\frac{1}{2}) + b$ $\quad$ 6

$f'(\frac{1}{2}) = -4$ $\quad b = 4$

$y = -4x + 4$

10. Find the derivative of the function $f(x) = 2x^2 + 1$ using the definition of the derivative.

$\frac{2(x+h)^2+1 - (2x^2+1)}{h} = \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{+1} - \cancel{2x^2} \cancel{-1}}{h}$

$= \frac{4xh + 2h^2}{h} = 4x + 2h$

$\lim_{h \to 0}(4x + 2h) = 4x$

11. $\frac{1}{3}\int_0^1 3x^2(x^3+1)^7\,dx =$ $\frac{(x^3+1)^8}{24}\Big|_0^1 = \frac{2^8}{24} - \frac{1}{24} = \frac{255}{24}$

12. $\int_1^2 (x^3 - x + \frac{1}{x^2})\,dx =$ $\frac{x^4}{4} - \frac{x^2}{2} - x^{-1}\Big|_1^2 = \frac{2^4}{4} - \frac{2^2}{2} - \frac{1}{2} - \left(\frac{1}{4} - \frac{1}{2} - 1\right)$

x^{-2}

$= 4 - 2 - \frac{1}{2} - \frac{1}{4} + \frac{3}{2} =$

$2 + 1 - \frac{1}{4} = \boxed{2\frac{3}{4}}$

13. $\int_{-3}^{-2} \frac{x^2-4}{x^2}\,dx =$ $\int_{-3}^{-2} x^0 - 4x^{-2}\,dx = \frac{x^2}{2} + 4x^{-1}\Big|_{-3}^{-2}$

$= \left(\frac{9}{2} - \frac{4}{3}\right) + \left(\frac{4}{2} - 2\right) = -\frac{5}{2} + 2 + \frac{4}{3}$

$= \frac{15+12-8}{6} = \frac{19}{6} = 3\frac{1}{6}$

14. $\int_1^2 (x+2)(x-1)\,dx =$ $\int_1^2 (x^2 + x - 2)\,dx$

$= \frac{x^3}{3} + \frac{x^2}{2} - 2x\Big|_1^2 = \frac{8}{3} + \frac{4}{2} - 4 - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$

$= \frac{7}{3} + \frac{3}{2} - 2 = \frac{14+9-12}{6} = \frac{11}{6}$

15. If $\int_5^{10} f(x)\,dx = 6$ and $\int_{10}^{15} (f(x) + 1)\,dx = 10$,

$\int_{10}^{15} f(x) = 10 - 5$

what is $\int_5^{15} f(x)\,dx$? $= \int_5^{10} + \int_{10}^{15} = 6 + 5 = 11$

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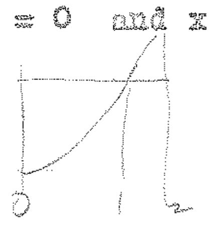
16. Find $\int_{-1}^{1} \sqrt{1-x^2}\,dx$ by finding the area under the curve geomerically
(Hint: Draw a picture)

$\frac{1}{2}\pi r^2 = \boxed{\frac{\pi}{2}}$

17. Find the area bounded by the curve $y = x^2$ and the line $y = 1$ between $x = 0$ and $x = 2$.



$1 - \int_0^1 x^2\,dx + \int_1^2 (x^2 - 1)$

$= \frac{x^3}{3}\Big|_0^1 + \frac{x^3}{3}\Big|_1^2 = -\frac{1}{3} + \frac{8}{3} - \frac{1}{3}$

$= \frac{8}{3} - \frac{2}{3} = \frac{6}{3} = 2$

18. Calculate the upper and lower sums for the function $f(x) = 4x^2 + 1$ over the interval $[0,2]$ using the partition

$$P = \{0, 1/2, 1, 3/2, 2\}.$$

$L(P) = 1(\frac{1}{2}) + 2(\frac{1}{2}) + 5(\frac{1}{2}) + 10(\frac{1}{2}) = \frac{18}{2} = 9$

$U(P) = 2(\frac{1}{2}) + 5(\frac{1}{2}) + 10(\frac{1}{2}) + 17(\frac{1}{2}) = \frac{34}{2} = 17$

19. Determine when the function $f(x) = -x^3 + x^2 + x + 1$ is increasing, decreasing. What are the maxima or minima?

$f'(x) = -3x^2 + 2x + 1 = (3x+1)(-x+1)$

$f'(x) > 0 \quad 3x+1>0 \quad -x+1>0$

$x > -\frac{1}{3} \quad x < 1$

$I\ (-\frac{1}{3}, 1)$

$f''(x) = -6x+2$

$x = -\frac{1}{3}$

$f''(-\frac{1}{3}) = 2+2 = 4$

$-\frac{1}{3}$ min

$f'(x) < 0 \quad 3x+1<0 \quad -x+1>0$

$x < -\frac{1}{3} \quad x<1$

$f''(1) = -6+2 =$

$x < -\frac{1}{3} \quad D(-\infty, -\frac{1}{3})\ (1, \infty)$

1 max

$x > -\frac{1}{3} \quad x > 1 \quad x > 1$

20. What should be the dimensions of the box with square bottom and no top with surface area 108 which has maximum volume?

$V(x) = x^2 y$

$x^2 + 4xy = 108$

$4xy = 108 - x^2$

$y = \frac{108 - x^2}{4x} = 27x^{-1} - \frac{x}{4}$

$V(x) = 27x - \frac{x^3}{4}$

$V'(x) = 27 - \frac{3}{4}x^2 = 0$

$\frac{3}{4}x^2 = 27$

$x^2 = \frac{27 \cdot 4}{3} = 9 \cdot 4 = 36$

$x = 6$

$y = \frac{27}{6} - \frac{6}{4} = \frac{54-18}{12} = \frac{36}{12}$

$6 \times 6 \times 3$