Math 131
Test II
Feb. 13, 1970

Name Key

Show work. Write DNE for "does not exist".

For problems 1–4 let

$$F(x) = \begin{cases} 2x+5, & x \neq 0 \\ 10, & x = 0 \end{cases} \qquad g(x) = \begin{cases} x^2-1, & x \neq 2 \\ A, & x = 2 \end{cases}$$

5 1. The function F is continuous at $x = 0$ TRUE( ) FALSE(✓)

5 2. In order for the function g to be continuous, A must be 3.

5 3. $\lim\limits_{x \to 2} F(x) =$ 9.

5 4. $\lim\limits_{x \to 1} F(x)g(x) =$ 0.

10 5. In the proof of $\lim\limits_{x \to 2} (3x-1) = 5$, if $\epsilon > 0$ is given, find a δ which satisfies the limit definition. Show work!

$$|3x-1-5| < \epsilon$$
$$|3x-6| < \epsilon$$
$$|x-2| < \epsilon/3 \qquad \delta = \epsilon/3$$

Find the indicated limits, if they exist.

10 6. $\lim\limits_{x \to 3} \dfrac{x^2+2x+1}{x^2-1} = \dfrac{3^2+2\cdot3+1}{3^2-1} = \dfrac{9+6+1}{8} = \dfrac{16}{8} = 2$

10 7. $\lim\limits_{x \to 3} \dfrac{x^2-9}{x-3} = \lim\limits_{x \to 3} \dfrac{(x-3)(x+3)}{x-3} = \lim\limits_{x \to 3} x+3 = 6$

10 8. $\lim\limits_{x \uparrow 0} \dfrac{|x|}{x} = \lim\limits_{x \uparrow 0} -1 = -1$

9. Find the derivative of the function $g(x) = \sqrt{x^2-2}$ using the definition of the derivative.

$$\frac{\sqrt{(x+h)^2-2} - \sqrt{x^2-2}}{h} = \frac{(x+h)^2 - 2 - x^2 + 2}{h\sqrt{(x+h)^2-2} + \sqrt{x^2-2}}$$

$$= \frac{x^2+2xh+h^2-x^2}{h\sqrt{\quad} + \sqrt{\quad}} = \frac{2x+h}{\sqrt{(x+h)^2-2}+\sqrt{x^2-2}}$$

$$g'(x) = \lim_{h \to 0} \frac{2x+h}{\sqrt{(x+h)^2-2}+\sqrt{x^2-2}} = \frac{2x}{2\sqrt{x^2-2}} = \frac{x}{\sqrt{x^2-2}}$$

10 10. Given that the derivative of $f(x) = \sqrt{x}$, $x \geq 0$ is $f'(x) = \frac{1}{2\sqrt{x}}$, $x > 0$; what is the derivative of $g(x) = \sqrt{x}\,(x^2+1)$?

$$g'(x) = \sqrt{x}(2x) + \frac{1}{2\sqrt{x}}(x^2+1) = \frac{5x^2+1}{2\sqrt{x}}$$

$$= 2x^{3/2} + \frac{1}{2}x^{3/2} + \frac{1}{2}x^{-1/2}$$

$$= 2\tfrac{1}{2}x^{3/2} + \frac{1}{2\sqrt{x}}$$

10 Differentiate the following functions.

11. $f(x) = x^3 + x^2 + 1 + \frac{2}{x} + \frac{3}{x^2}$ $\quad 2x^{-1} + 3x^{-2}$

$$f'(x) = 3x^2 + 2x - 2x^{-2} - 6x^{-3}$$

$$= 3x^2 + 2x - \frac{2}{x^2} - \frac{6}{x^3}$$

10 12. $h(x) = \frac{x^4 - x^3}{x^3 - 1}$

$$h'(x) = \frac{(x^3-1)(4x^3-3x^2) - (x^4-x^3)(3x^2)}{(x^3-1)^2}$$

$$= \frac{4x^6 - 3x^5 - 4x^3 + 3x^2 - 3x^6 + 3x^5}{(x^3-1)^2} = \frac{x^6 - 4x^3 + 3x^2}{(x^3-1)^2}$$

$$\frac{x^2(x^4 - 4x + 3)}{(x^3-1)^2}$$

$$x-1 \overline{) x^4 - 4x + 3} \quad x^3 + x^2 + x - 3$$

$x^4 - x^3$

$x^3 \; x^2$