Math 151
Test III
March 4, 1970

Name Key

Show work! Use back for scratch paper. All problems worth 10 points.

1. $D^2(x^3 + x^2 + \frac{1}{x} - \frac{3}{x^3}) = D(3x^2 + 2x - x^{-2} + 9x^{-4})$

$= 6x + 2 + 2x^{-3} - 36x^{-5}$

$= 6x + 2 + \frac{2}{x^3} - \frac{36}{x^5}$

2. $D(x - D(6 - \frac{1}{x})) = D(x - (x^{-2})) = 1 + 2x^{-3}$

$= 1 + \frac{2}{x^3}$

3. If $h(x) = (x^3 + x)^{10}$, find $h'(x)$.

$h'(x) = 10(x^3 + x)^9(3x^2 + 1)$

4. If $f'(x) = x^3 + \frac{x^2}{2}$, and $f(2) = 14$, then find $f(x)$.

$f(x) = \frac{x^4}{4} + \frac{x^3}{6} + C$

$14 = \frac{16}{4} + \frac{8}{6} + C$

$C = 10 - \frac{4}{3} = \frac{26}{3}$

$f(x) = \frac{x^4}{4} + \frac{x^3}{6} + \frac{26}{3}$

$= 8\frac{2}{3}$

5. Find the equation of the straight line which is tangent to the curve

$$y = x^3 + 2x^2 + x + 1$$

at the point (2,19).

$f'(x) = 3x^2 + 4x + 1$ 3

$m = f'(2) = 12 + 8 + 1 = 21$

$y = 21x - 23$

$y = 21x + b$

$19 = 21(2) + b \qquad b = -19 - 23$

6. On what interval(s) is the function $f(x) = x + \frac{1}{x}$ $= x + x^{-1}$ increasing? decreasing?

$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2} > 0$

$< 0 \qquad 1 < \frac{1}{x^2} \qquad\qquad 1 > \frac{1}{x^2} \qquad x^2 > 1$

$x^2 < 1 \qquad\qquad\qquad x > 1,\ x < -1$

$-1 < x < 1$

Decreasing $(-1,0), (0,1)$, Increasing, $(-\infty,-1), (1,\infty)$

7. On what intervals is the function $g(x) = \frac{x}{1 + x}$ concave up? concave down?

$g'(x) = \frac{(1+x) - x}{(1+x)^2} = \frac{1}{(1+x)^2} = (1+x)^{-2}$ 0

$g''(x) = -2(1+x)^{-3}$

$= \frac{-2}{(1+x)^3}$ 6 $> 0 \quad 1+x < 0 \qquad x < -1$

$< 0 \quad 1+x > 0 \qquad x > -1$

cc↑ $(-\infty,-1)$, cc↓ $(-1,\infty)$

8. Consider

$$h(x) = \frac{x}{1 + x^2}.$$

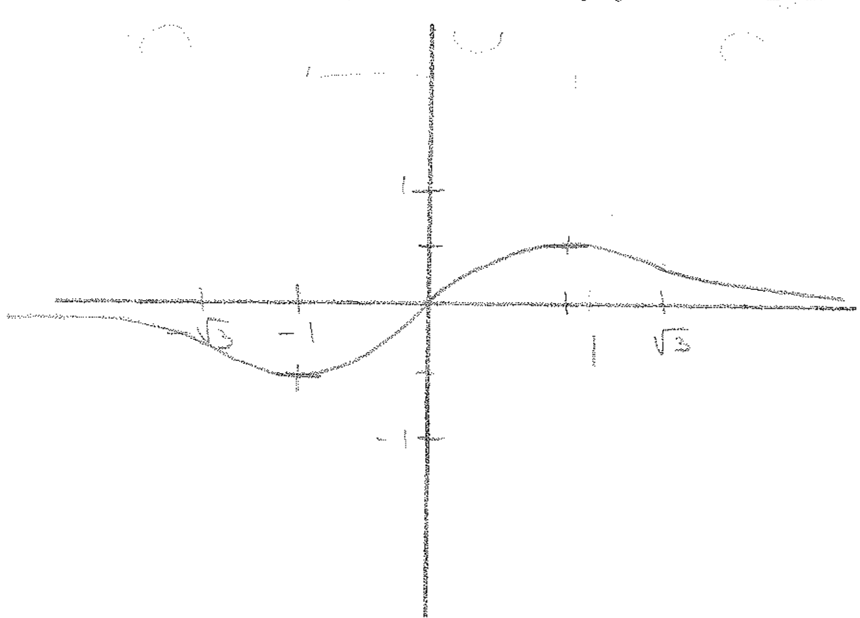
Sketch a careful graph of this function with the following information. Plot all critical points and points of inflection, if any, and label them.

$h'(x) > 0$ for $-1 < x < 1$ $\qquad$ $h''(x) > 0$ for $x > \sqrt{3}$ or $-\sqrt{3} < x < 0$

$h'(x) < 0$ for $x > 1$ or $x < -1$ $\qquad$ $h''(x) < 0$ for $0 < x < \sqrt{3}$, or $x < -\sqrt{3}$

$h'(x) = 0$ for $x = 1, -1.$ $\qquad$ $h''(x) = 0$ for $x = 0, \sqrt{3}, -\sqrt{3}$

$\frac{\sqrt{3}}{1+3} = \frac{\sqrt{3}}{4}$



9. Calculate the upper and lower sums for the function $f(x) = 2x - 1$ over $[1,2]$ using the partition

$$P = \{1,\ 1\tfrac{1}{4},\ 1\tfrac{1}{2},\ 1\tfrac{3}{4},\ 2\}.$$

What would you predict for the value of $\int_1^2 f(x)\,dx$?

$$L(P) = \overset{3}{(2-1)}\left(\tfrac{1}{4}\right) + \left(2\tfrac{1}{2}-1\right)\left(\tfrac{1}{4}\right) + (3-1)\left(\tfrac{1}{4}\right) + \left(3\tfrac{1}{2}-1\right)\left(\tfrac{1}{4}\right)$$

$$= \frac{1}{4} + \frac{3}{2}\cdot\frac{1}{4} + \frac{2}{4} + \frac{5}{2}\cdot\frac{1}{4}$$

$$= \frac{2+3+4+5}{8} = \frac{14}{8} = \boxed{\frac{7}{4}}$$

$$U(P) = \left(2\tfrac{1}{2}-1\right)\left(\tfrac{1}{4}\right) + (3-1)\left(\tfrac{1}{4}\right) + \left(3\tfrac{1}{2}-1\right)\left(\tfrac{1}{4}\right) + (4-1)\left(\tfrac{1}{4}\right)$$

$$= \frac{3}{2}\cdot\frac{1}{4} + \frac{2}{4} + \frac{5}{2}\cdot\frac{1}{4} + \frac{3}{4} = \frac{3+4+5+6}{8} = \frac{18}{8} = \boxed{\frac{9}{4}}$$

$$\int_1^2 f(x)\,dx = 2$$

10. A bullit is shot directly upwards with a muzzle velocity of 1600 ft./sec.. Neglecting air friction, how high will it go before it falls back to earth?

$$a(t) = -32$$

$$v(t) = -32t + C$$

$$1600 = v(0) = -32(0) + C$$

$$C = 1600$$

$$v(t) = -32t + 1600$$

$$s(t) = -16t^2 + 1600t + C$$

$$0 = s(0) = C$$

$$s(t) = -16t^2 + 1600t$$

$$-32t + 1600 = 0$$

$$t = \frac{1600}{32} = 50$$

$$s(50) = -16(2500) + 1600(50)$$

$$= 1600(-25 + 50) = 25(1600)$$

$$= 40{,}000 \text{ ft.}$$