Math 231
Test I
April 17, 1970

Name KEY

Show work for full credit!

Part I: 5 points each.

1. What restriction must be placed on x so that log log x is defined?

$\log x > 0$, so $x > 1$

2. Solve for x: $\log(2x+3) = 2 \log x$

$$2x+3 = x^2$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$

$x = -1, 3$ (−1 marked X, 3 checked)

3. Simplify:

$\log e^e = e$

Part II: Differentiate the following. 10 points each.

4. $f(x) = 5^x + e^e + x^{4/3} = e^{x \log 5} + e^e + x^{4/3}$

$$f'(x) = e^{x \log 5} \log 5 + 0 + \tfrac{4}{3} x^{1/3}$$
$$= 5^x \log 5 + \tfrac{4}{3} x^{1/3}$$

5. $g(x) = \log((x+e^x)^2) = 2 \log (x+e^x)$

$$g'(x) = \frac{2}{x+e^x}(1+e^x)$$

or:

$$g'(x) = \frac{1}{(x+e^x)^2} 2(x+e^x)(1+e^x)$$
$$= \frac{2(1+e^x)}{x+e^x}$$

6. $h(x) = ((x^2+e)^\pi + x)^{3/2}$

$$h'(x) = \frac{3}{2}\left((x^2+e)^\pi + x\right)^{1/2}\left(\pi(x^2+e)^{\pi-1}(2x) + 1\right)$$

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Part III: Integrate the following. 10 points each.

7. $\displaystyle\int_1^e \frac{(\log x)^2}{x}\,dx = \frac{(\log x)^3}{3}\Big|_1^e = \frac{1}{3}$

8. $\int_1^2 x^3 \log x \, dx = \frac{x^4 \log x}{4}\Big|_1^2 - \int_1^2 \frac{x^3}{4} = \frac{16\log 2}{4} - 0 - \left(\frac{x^4}{16}\right)\Big|_1^2$

$f(x) = \log x \quad g'(x) = x^3$

$f'(x) = \frac{1}{x} \quad g(x) = \frac{x^4}{4}$

$= 4\log 2 - \left(\frac{16}{16} - \frac{1}{16}\right)$

$= 4\log 2 - \frac{15}{16}$

9. $\int_1^2 x(1+x)^{5/2}\,dx = \frac{2x(1+x)^{7/2}}{7}\Big|_1^2 - \int_1^2 \frac{2}{7}(1+x)^{7/2}\,dx$

$f(x) = x \quad g'(x) = (1+x)^{5/2}$

$f'(x) = 1 \quad g(x) = \frac{2}{7}(1+x)^{7/2}$

$= \frac{4(3)^{7/2}}{7} - \frac{2\cdot 2^{7/2}}{7} - \frac{4}{63}(1+x)^{9/2}\Big|_1^2$

$= \frac{4(3)^{7/2}}{7} - \frac{16\sqrt{2}}{7} - \frac{4(3)^{9/2}}{63} + \frac{4\cdot 2^{9/2}}{63}$

$= \frac{4\cdot 27\sqrt{3}}{7} - \frac{16\sqrt{2}}{7} - \frac{4\cdot 81\sqrt{3}}{63} + \frac{64\sqrt{2}}{63}$

$= \frac{972\sqrt{3} - 144\sqrt{2} - 244\sqrt{3} + 64}{63}$

$= \frac{828\sqrt{3} - 80\sqrt{2}}{63}$

10. $\frac{1}{2}\int_0^1 2x e^{x^2}\,dx = \frac{1}{2}e^{x^2}\Big|_0^1 = \frac{1}{2}e - \frac{1}{2} = \frac{e-1}{2}$

Part III: 15 points.

11. Let $f(x) = x e^x$. Where is f increasing? decreasing? concave up? concave down? Where are local maxima or minima? points of inflection? if any. Draw sketch.

$f'(x) = xe^x + e^x = (x+1)e^x$

$f''(x) = xe^x + e^x + e^x = (x+2)e^x$

$f'(x) > 0$	$f'(x) < 0$	$f''(x) > 0$	$f''(x) < 0$
$x > -1$	$x < -1$	$x > -2$	$x < -2$
I	D	cc ↑	cc ↓

$x = -1$ local min

$x = -2$ pt of inflection