Math 231
Test I
Oct. 12, 1973

Name KEY

Show work for full credit!

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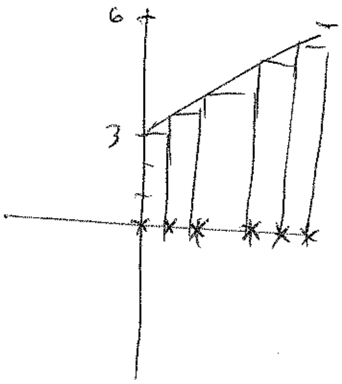
(5) 1. State precisely the Fundamental Theorem of Calculus.

If $f(x)$ is cont for all x in $[a,b]$ and $F(x)$ is any antiderivative of f in $[a,b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

(7) 2. For the given function and partition, write out (but do not do the arithmetic) the upper and lower sums.

a. $f(x) = x+3$, x in $[0,3]$ $P = \{0, 1/2, 1, 2, 5/2, 3\}$.

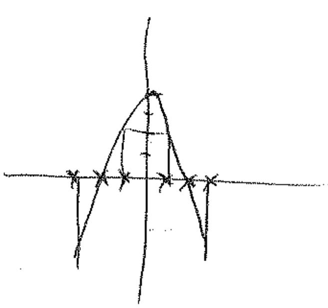


$$S(P) = \underset{3}{(0+3)}\left(\tfrac{1}{2}\right) + \underset{3\frac{1}{2}}{\left(\tfrac{1}{2}+3\right)}\left(\tfrac{1}{2}\right) + \underset{4}{(1+3)}\cdot 1 + \underset{5}{(2+3)}\left(\tfrac{1}{2}\right) + \underset{5\frac{1}{2}}{\left(\tfrac{5}{2}+3\right)}\cdot\tfrac{1}{2}$$

$$T = \underset{3\frac{1}{2}}{\left(\tfrac{1}{2}+3\right)}\cdot\tfrac{1}{2} + \underset{4}{(1+3)}\tfrac{1}{2} + \underset{5}{(2+3)}(1) + \underset{5\frac{1}{2}}{\left(\tfrac{5}{2}+3\right)}\left(\tfrac{1}{2}\right) + \underset{6}{(3+3)}\left(\tfrac{1}{2}\right)$$

no 0 -2

b. $f(x) = 4-x^2$, x in $[-3,3]$, $P = \{-3, -2, -1, 1, 2, 3\}$.



$$S(P) = \underset{-5}{\left(4-(-3)^2\right)}\cdot 1 + \underset{0}{\left(4-(-2)^2\right)}\cdot 1 + \underset{3}{\left(4-(-1)^2\right)}\cdot 2 + \underset{0}{\left(4-2^2\right)}\cdot 1 + \underset{-5}{\left(4-3^2\right)}\cdot 1$$

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$$T(P) = \underset{0}{\left(4-(-2)^2\right)}\cdot 1 + \underset{3}{\left(4-(-1)^2\right)}\cdot 1 + \underset{4}{\left(4-0^2\right)}\cdot 2 + \underset{3}{\left(4-1^2\right)}(\cdot 1) + \underset{0}{\left(4-2^2\right)}\cdot 1$$

(10) 3. True or False?

a. $\int \frac{2x}{2x+1}\, dx = 2x+1 + C$

NO $\quad D(2x+1) = 2$

b. $\int (x^2-6)^{10} dx = \frac{(x^2-6)^{11}}{11} + C$

$D\frac{(x^2-6)^{11}}{11} = \frac{11(x^2-6)^{10}}{\cancel{11}} \cdot 2x$ NO

(10) 4. Write out:

a. $\sum_{i=1}^{5} \frac{i+1}{2} = \frac{1+1}{2} + \frac{2+1}{2} + \frac{3+1}{2} + \frac{4+1}{2} + \frac{5+1}{2}$

b. $\sum_{k=3}^{7} \frac{k+1}{2^k} = \frac{3+1}{2^3} + \frac{4+1}{2^4} + \frac{5+1}{2^5} + \frac{6+1}{2^6} + \frac{7+1}{2^7}$

(48) 5. Evaluate each of the following:

(8ea)

a. $\int_0^1 x^3+6x+5\, dx = \frac{x^4}{4} + \frac{6x^2}{2} + 5x \Big|_0^1 = \frac{x^4}{4} + 3x^2 + 5x \Big|_0^1 = \frac{1}{4} + 3 + 5 - 0$

$= 8\frac{1}{4}$

b. $\int_1^4 x\sqrt{x}\, dx = \int_1^4 x^{3/2} dx = \frac{x^{5/2}}{5/2}\Big|_1^4 = \frac{2}{5}(4^{5/2} - 1^{5/2})$

$= \frac{2}{5}(2^5 - 1) = \frac{2}{5} 31 = \frac{62}{5}$

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c. $\displaystyle\int_{-2}^{-1}(x+5)^6\,dx = \frac{(x+5)^7}{7\,\cancel{6x}}\Big|_{-2}^{-1} = \frac{4^7}{7} - \frac{3^7}{7}$ ok

d. $\displaystyle\int_{1}^{3} x(2x+5)\,dx = \int_1^3 2x^2+5x\,dx = \frac{2x^3}{3}+\frac{5x^2}{2}\Big|_1^3$

$$= \frac{2\cdot 3^3}{3} + \frac{5(3)^2}{2} - \left(\frac{2}{3}+\frac{5}{2}\right)$$

$$= 18 + \frac{45}{2} - \frac{2}{3} - \cancel{\frac{5}{2}}$$

$$= 38 - \frac{2}{3} = 37\tfrac{1}{3}$$

e. $\displaystyle\int_{4}^{13}\frac{1}{\sqrt{3x-3}}\,dx = \int_4^{13}(3x-3)^{-1/2}\,dx = \frac{2}{1}\cdot\frac{1}{3}\,\frac{(3x-3)^{1/2}}{1/2}\Big|_4^{13}$

$$= \frac{2\,36^{1/2}}{3\,\cancel{6}} - \frac{2\,9^{1/2}}{3\,\cancel{6}} = \frac{2}{3}\cancel{\left(\frac{6}{6}-\frac{3}{6}\right)}$$

$$= \cancel{1-\frac{1}{2}=\frac{1}{2}}$$

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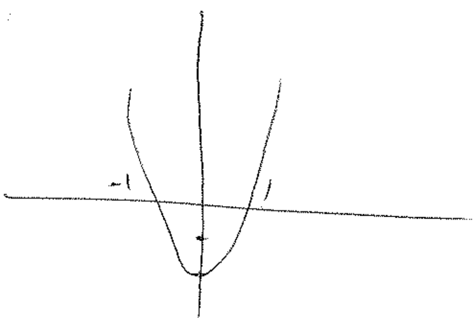
5 e. $\frac{1}{2}\int (x^2+6)^7 2x \, dx =$ $\frac{(x^2+6)^8}{2\cdot 8} \cancel{+C} = \frac{(x^2+6)^8}{16} + C$ 2

(10) 6. Find the area ~~bbb~~ of the region bouded by the curves $y = x^2-1$ and the x-axis.



$x = \pm 1$

$$-\int_{-1}^{1} x^2 - 1 \, dx = -\left(\frac{x^3}{3} - x\right)\Big|_{-1}^{1}$$

$$= -\left(\frac{1}{3} - 1\right) + \left(\frac{-1}{3} - (-1)\right)$$

$$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$