Math 211
Test II
November, 2, 1973

Name KEY

Show work for full credit!

15 1. Quick answer, no partial credit:

a. $\int_3^4 \frac{1}{x}\,dx = \ln x \Big|_3^4 = \ln 4 - \ln 3$

b. $\int e^{-x}\,dx = -e^{-x} + C$

c. If $y = e^{-5}$, then $\ln y = -5$

d. $\ln(\ln e) = \ln 1 = 0$

e. The derivative of $f(x) = \ln 5$ is 0

10 2. Complete the following definitions:

a. $\ln x = \int_1^x \frac{1}{t}\,dt \quad x > 0$

b. $a^x = e^{x \ln a}$

3. Find the derivative of each of the following

5 a. $x \cdot e^x + e^3$

-2

$x e^x + 1 e^x + 0$

$= x e^x + e^x = (x+1)e^x$

5 b. e^{x^3+3}

-1

$e^{x^3+3}(3x^2 \cdot 4)$

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2 3 c. $\ln(2x^2+3)$ $\qquad \frac{1}{2x^2+3} \cdot 4x$

5 d. $\sqrt{3+e^{2x}}$ $\qquad \frac{1}{2}(3+e^{2x})^{-1/2} e^{2x} \cdot 2 = \frac{e^{2x}}{\sqrt{3+e^{2x}}}$

10 4. Use the trapezoidal rule with 5 subintervals to approximate $\int_1^2 \frac{2}{x+1}\,dx$. (Set up, do not do arithmetic).

1 1.2 1.4 1.6 1.8 2

$\frac{.2}{2}\left[\frac{2}{1+1} + \frac{2}{1+(1.2)} + \frac{2}{1+1.4} + \frac{2}{1+1.6} + \frac{2}{1+1.8} + \frac{2}{1+2}\right]$

$\frac{1}{5}\left[\frac{1}{2}\cdot 1 + \frac{10}{11} + \frac{10}{12} + \frac{10}{13} + \frac{10}{14} + \frac{1}{3}\right]$

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5. Integrate the following:

9 a. $\int \frac{x^2+1}{x-1}\,dx = \int x+1+\frac{2}{x-1}\,dx = \frac{x^2}{2}+x+2\ln|x-1|+C$

$$\begin{array}{r} x+1 \\ x-1\,\overline{)\,x^2+1} \\ \underline{x^2-x} \\ x+1 \\ \underline{x-1} \\ 2 \end{array}$$

$u = x-1$

$du = dx$

$$\int \frac{(u+1)^2+1}{u}\,du = \int \frac{u^2+2u+1+1}{u}\,du$$

$$= \int u+2+\frac{2}{u}\,du = \frac{u^2}{2}+2u+2l$$

$$= \frac{(x-1)^2}{2}+2(x-1)+2\ln|x-1|+$$

9 b. $\int_1^3 \frac{(\ln x)^2}{x}\,dx = \int_0^{\ln 3} u^2\,du \quad \frac{u^3}{3}\Big|_0^{\ln 3} = \frac{(\ln 3)^3}{3} \cancel{+C}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\frac14\left[\frac{u^{5/2}}{5/2}+\frac{u^{3/2}}{3/2}\right]\Bigg|_1^3$$

9 c. $\int_1^2 x\sqrt{2x-1}\,dx = \frac12\int_1^3 \frac{u+1}{2}\sqrt{u}\,du = \frac14\int_1^3 (u^{3/2}+u^{1/2})\,du$

$u = 2x-1$

$du = 2\,dx$

$x = \frac{u+1}{2}$

$\frac{1}{10}3^{5/2}+\frac16$

$$\boxed{\begin{gathered} \frac{1}{10}u^{5/2}+\frac16 u^{3/2}\Big|_1^3 \\ \frac{1}{10}3^{5/2}+\frac16 3^{3/2}-\left[\frac{1}{10}+\frac16\right] \quad 6+10=\frac{16}{60} \\ \frac{9\sqrt3}{10}+\frac{3\sqrt3}{6}+\frac{16}{60} \\ +\frac{24}{10}\sqrt3+\frac{16}{60} = 3.89 \end{gathered}}$$

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$$\frac52 3^{5/2} - \frac32 3^{3/2} - \left[\frac52-\frac32\right]$$

$$= \frac52 9\sqrt3 - \frac32 3\sqrt3 - 1$$

$$= \frac{45}{2}\sqrt3 - \frac92\sqrt3 - 1 = 18\sqrt3 - 1$$

d. $\frac{1}{3}\int 3x^2 e^{x^3-3}\,dx$ $\quad \frac{1}{3}\int e^{x^3 - 3}\,\cancel{dx} + C$

10 6. Assume that the rate of growth of a rabbit population is proportional to the size of the population. If the population increases from 400 to 900 in two years, what is the population after 3 years?

$$y = Ce^{kt}$$

$t = 0 \quad y = 400$

$$400 = Ce^{0} \qquad C = 400$$

$t = 2 \quad y = 900$

$$900 = 400e^{k2}$$

-.5 $\quad e^{k2} = \frac{9}{4}$

$$k2 = \ln\frac{9}{4} \qquad k = \frac{1}{2}\ln\frac{9}{4} = \ln\frac{3}{2}$$

$$y = 400e^{(\ln\frac{3}{2})t} = 400\left(\frac{3}{2}\right)^t$$

-2

$t = 3$

$$y = 400\left(\frac{3}{2}\right)^3 = 100 \cdot \frac{27}{2} = 1350$$

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