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Math 231
Test III
Nov. 20, 1973

Name KEY

Show work for full credit!

10 1. Derive the formula for $D\sin x$.

$$D\sin x = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \frac{\sin h}{h} \right]$$

$$= \sin x \underbrace{\lim_{h \to 0} \left(\frac{\cos h - 1}{h} \right)}_{0} + \cos x \underbrace{\lim_{h \to 0} \frac{\sin h}{h}}_{1}$$

$$= \cos x$$

10 2. Evaluate:

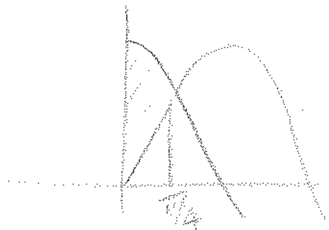
a. $\tan \frac{7\pi}{4} = -1$

b. $\sin(\sin^{-1} \frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{2}$

c. $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$

d. $\tan^{-1}(\tan \frac{3\pi}{4}) = \;\; = -\frac{\pi}{4}$

10 3. Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, and the y-axis.



$$\int_0^{\pi/4} \cos x \, dx - \int_0^{\pi/4} \sin x \, dx = \sin x \Big|_0^{\pi/4} + \cos x \Big|_0^{\pi/4}$$

$$= \frac{1}{\sqrt{2}} - 0 + \frac{1}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

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4. Find the following derivatives:

a. $f(x) = \sin 3x$, $f'(x) =$ $\cos 3x \cdot 3$

b. $D(\cos x \sin 2x) =$ $\cos x \cos 2x \cdot 2 + \sin 2x(-\sin x)$

$= 2\cos x \cos 2x - \sin 2x \sin x$

c. $D \tan^{-1}(x^2) =$ $\dfrac{1}{1+(x^2)^2} \, 2x = \dfrac{2x}{1+x^4}$

d. $D \cot(2x-1) =$ $-\csc^2(2x-1) \cdot 2$

e. $D((1+\sec x)^3) =$ $3(1+\sec x)^2 \sec x \tan x$

5. Evaluate the following integrals:

7 a. $\displaystyle\int_0^{\pi/4} \cos 4x \, dx =$ $\left.\dfrac{\sin 4x}{4}\right|_0^{\pi/4} = \dfrac{\sin \pi}{4} - \sin 0$

$= 0$

8 **5b.** $\int_{\pi/6}^{\pi/3} (\sin 2x)^3\,dx = \int_{\pi/6}^{\pi/3} \sin^2 2x \ \sin 2x\,dx$

$$= \int_{\pi/6}^{\pi/3} (1-\cos^2 2x)\sin 2x\,dx = -\int_{1/2}^{-1/2} \frac{1-u^2}{2}\,du = -\left(\frac{u}{2}-\frac{u^3}{6}\right)\Big|_{1/2}^{-1/2}$$

$u = \cos 2x \quad x=\pi/6 \quad u=\cos\frac{\pi}{3}=\frac{1}{2}$

$du = -\sin 2x \cdot 2 \quad x=\pi/3 \quad u=\cos\frac{2\pi}{3}=-\frac{1}{2}$

$$= -\left(-\frac{1}{4}+\frac{1}{48}\right)+\left(\frac{1}{4}-\frac{1}{48}\right) = \frac{1}{2}-\frac{1}{24}=\frac{11}{24}$$

8 **c.** $\int \frac{1}{x^2+4x+8}\,dx = \int \frac{1}{x^2+4x+4+4}\,dx$

$$= \int \frac{1/4}{\left(\frac{x+2}{2}\right)^2+1}\,dx = \frac{1}{2}\int \frac{1}{u^2+1}\,du = \frac{1}{2}\tan^{-1} u + C$$

$u = \frac{x+2}{2}$ $\qquad = \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+C$

$du = \frac{1}{2}\,dx$

8 **d.** $\int \frac{2\sec x\tan x}{1+\sec x}\,dx = 2\int \frac{1}{u}\,du = 2\ln|u|+C$

$u = 1+\sec x$ $\qquad = 2\ln|1+\sec x|+C$

$du = \sec x\tan x\,dx$

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6 5a. $\int_0^1 \frac{1+x}{1+x^2}\,dx =$ (Hint: split into sum of two fractions) $\int_0^1 \frac{1}{1+x^2}\,dx + \int_0^1 \frac{x}{1+x^2}\,dx$

$$= \tan^{-1}x \Big|_0^1 + \tfrac{1}{2}\ln(1+x^2)\Big|_0^1$$

$$= \tan^{-1}1 - \tan^{-1}0 + \tfrac{1}{2}\ln 2 - \tfrac{1}{2}\ln 0$$

$$= \frac{\pi}{4} + \frac{1}{2}\ln 2$$

8 2. $\int 2x \sin 3x\,dx =$ (Hint: try integration by parts) 27

$u = 2x \quad dv = \sin 3x\,dx$

$du = 2\,dx \quad v = -\frac{\cos 3x}{3}$

$$= -2x\,\frac{\cos 3x}{3} - \int -\frac{2}{3}\cos 3x\,dx$$

$$= -\frac{2}{3}x\cos 3x + \frac{2}{3}\,\frac{\sin 3x}{3} + C = -\frac{2}{3}x\cos 3x + \frac{2}{9}\sin 3x + C$$

8 8. $\int x(x+3)^3\,dx =$ 28

$x(x^3 + 3x^2\cdot 3 + 3x\cdot 3^2 + 3^3)$

$x^4 + 9x^3 + 27x^2 + 27x$

$u = x \quad dv = (x+3)^3\,dx$

$du = dx \quad v = \frac{(x+3)^4}{4}$

$$= x\,\frac{(x+3)^4}{4} - \int \frac{(x+3)^4}{4}\,dx$$

$$= \frac{x(x+3)^4}{4} - \frac{(x+3)^5}{20} + C$$

$u = x+3 \quad x = u-3$

$du = dx$

$$\int (u-3)u^3\,du$$

$$= \int u^4 - 3u^3\,du$$

$$= \frac{u^5}{5} - \frac{3u^4}{4} + C$$

$$= \frac{(x+3)^5}{5} - \frac{3(x+3)^4}{4} + C$$

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