

(10)

1. Complete the following definitions precisely:

- a. $\ln x = \int_1^x \frac{1}{t} dt, x > 0$
- b. $y = e^x$ iff $x = \ln y$

30)

2. Quick answer. No partial Credit.

- a. $\ln(\ln e) = \ln 1 = 0$
- b. $\log_3 9 = 2$
- c. $D \ln 2 = 0$
- d. $D e^x = e^x$
- e. If $\log_{10} x = -2$, then $x = 10^{-2} = \frac{1}{100}$
- f. $\int e^{2x} dx = \frac{e^{2x}}{2} + C$
- g. $\int \frac{1}{x+1} dx = \ln|x+1| + C$
- h. $\ln(x^2-1) = 2$, solve for x . $\pm \sqrt{1+e^2}$
- i. $D((\ln 3)x) = \ln 3$
- j. $D \ln x^2 = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

$$\begin{aligned} x^2 - 1 &= e^2 \\ x^2 &= 1 + e^2 \\ x &= \pm \sqrt{1 + e^2} \end{aligned}$$

10

3. Write out (do not compute) the approximation given by the trapezoidal rule with 5 subintervals for the integral

$\{1, 1.2, 1.4, 1.6, 1.8, 2\}$ $\int_1^2 \frac{2}{x} dx$

$$\frac{2-1}{5} \left\{ \frac{1(2)}{2(1)} + \frac{2}{1.2} + \frac{2}{1.4} + \frac{2}{1.6} + \frac{2}{1.8} + \frac{1}{2} \left(\frac{2}{2} \right) \right\}$$

or $\left\{ 1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2 \right\}$

$$\frac{2-1}{5} \left\{ \frac{1}{2} \cdot \frac{2}{1} + 2\left(\frac{5}{6}\right) + 2\left(\frac{5}{7}\right) + 2\left(\frac{5}{8}\right) + 2\left(\frac{5}{9}\right) + \frac{2}{2 \cdot 2} \right\}$$

4. Find the derivative of each of the following functions:

a. $f(x) = \ln(x^4 - 2)$ $f'(x) = \frac{1}{x^4 - 2} \cdot 4x^3$
 $= \frac{4x^3}{x^4 - 2}$

b. $f(x) = \sqrt{e^x + 2} = (e^x + 2)^{1/2}$
 $f'(x) = \frac{1}{2}(e^x + 2)^{-1/2} \cdot e^x$
 $= \frac{e^x}{2\sqrt{e^x + 2}}$

c. $g(x) = e^{3x-5}$
 $g'(x) = e^{3x-5} \cdot 3 = 3e^{3x-5}$

d. $g(x) = x^2 \ln x$
 $g'(x) = 2x \ln x + \frac{x^2}{x}$
 $= 2x \ln x + x$

5. Find each of the following integrals:

(10) a. $\int_0^1 \frac{8x}{4x^2 + 5} dx = \frac{1}{8} \ln(4x^2 + 5) \Big|_0^1$
 $= \frac{1}{8} \ln 9 - \frac{1}{8} \ln 5$

$$5. b. \int_1^e \frac{(\ln x)^3}{x} dx =$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int_{x=1}^{x=e} u^3 du = \frac{u^4}{4} \Big|_{x=1}^{x=e}$$

$$= \frac{(\ln x)^4}{4} \Big|_1^e$$

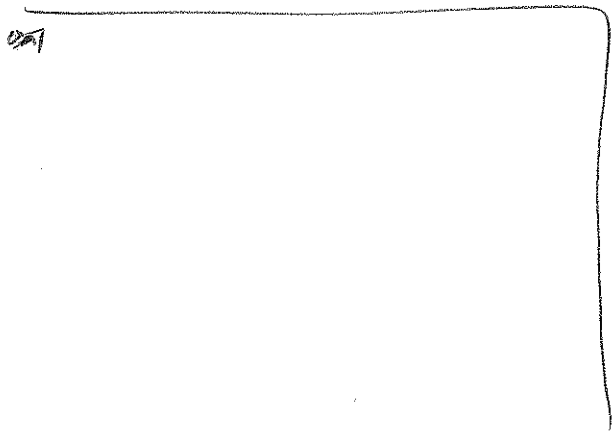
$$= \frac{(\ln e)^4}{4} - \frac{(\ln 1)^4}{4}$$

$$= \frac{1}{4} - 0$$

$$\text{or } \frac{u^4}{4} \Big|_0^1$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$



(10)

$$\int 4x \sqrt{2x+1} dx = \int (u-1) \sqrt{u} du$$

$$u = 2x+1$$

$$du = 2 dx$$

$$2x = u-1$$

$$= \int u^{3/2} - u^{1/2} du$$

$$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2} + C$$