Math 231
Test II
May 3, 1974

Name KEY

(10) 1. Complete the following definitions precisely:

a. $\ln x = \int_1^x \frac{1}{t}\,dt, \ x>0$

b. $y = e^x$ iff $x = \ln y$

30) 2. Quick answer. No partial Credit.

a. $\ln(\ln e)) = \ln 1 = 0$

b. $\log_3 9 = 2$

c. $D \ln 2 = 0$

d. $D e^x = e^x$

e. If $\log_{10} x = -2$, then $x = 10^{-2} = \frac{1}{100}$

f. $\int e^{2x}\,dx = \frac{e^{2x}}{2} + C$

g. $\int \frac{1}{x+1}\,dx = \ln|x+1| + C$

$x^2 - 1 = e^2$

$x^2 = 1 + e^2$

$x = \pm\sqrt{1+e^2}$

h. $\ln(x^2-1) = 2$, solve for x. $\pm\sqrt{1+e^2}$

i. $D((\ln 3)x) = \ln 3$

j. $D \ln x^2 = \frac{1}{x^2}\cdot 2x = \frac{2}{x}$

10 3. Write out (do not compute) the approximation given by the trapezoidal rule with 5 subintervals for the integral $\int_1^2 \frac{2}{x}\,dx$.

$\{1, 1.2, 1.4, 1.6, 1.8, 2\}$

$$\frac{2-1}{5}\left\{\frac{1}{2}\left(\frac{2}{1}\right) + \frac{2}{1.2} + \frac{2}{1.4} + \frac{2}{1.6} + \frac{2}{1.8} + \frac{1}{2}\left(\frac{2}{2}\right)\right\}$$

or $\{1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2\}$

$$\frac{2-1}{5}\left\{\frac{1}{2}\cdot\frac{2}{1} + 2\left(\frac{5}{6}\right) + 2\left(\frac{5}{7}\right) + 2\left(\frac{5}{8}\right) + 2\left(\frac{5}{9}\right) + \frac{2}{2\cdot 2}\right\}$$

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4. Find the derivative of each of the fillowing functions:

a. $f(x) = \ln(x^4-2)$ $\qquad f'(x) = \frac{1}{x^4-2}\cdot 4x^3$

$$= \frac{4x^3}{x^4-2}$$

b. $f(x) = \sqrt{e^x+2}$ $= (e^x+2)^{1/2}$

$$f'(x) = \frac{1}{2}(e^x+2)^{-1/2}\cdot e^x$$

$$= \frac{e^x}{2\sqrt{e^x+2}}$$

c. $g(x) = e^{3x-5}$

$$g'(x) = e^{3x-5}\cdot 3 \quad = 3e^{3x-5}$$

d. $g(x) = x^2 \ln x$

$$g'(x) = 2x\ln x + \frac{x^2}{x}$$

$$= 2x\ln x + x$$

5. Find each of the following integrals:

(10) a. $\frac{1}{8}\int_0^1 \frac{8x}{4x^2+5}\,dx = \frac{1}{8}\ln(4x^2+5)\Big|_0^1$

$$= \frac{1}{8}\ln 9 - \frac{1}{8}\ln 5$$

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5. b. $\int_1^e \frac{(\ln x)^3}{x}\,dx = \int_{x=1}^{x=e} u^3\,du = \left.\frac{u^4}{4}\right|_{x=1}^{x=e}$ or $\left.\frac{u^4}{4}\right|_0^1$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \left.\frac{(\ln x)^4}{4}\right|_1^e$$

$$= \frac{(\ln e)^4}{4} - \frac{(\ln 1)^4}{4}$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$

(10) c. $\int 4x\sqrt{2x+1}\,dx = \int (u-1)\sqrt{u}\,du$

$u = 2x+1$

$du = 2dx$

$2x = u-1$

$$= \int u^{3/2} - u^{1/2}\,du$$

$$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2} + C$$