Math 231
Test III
May 24, 1974 Name KEY

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3 ea I. Short answer. No partial credit.

1. If $f'(x) = k \cdot f(x)$, then $f(x) =$ ce^{kx}

2. If $\tan^{-1} x = \frac{\pi}{4}$, then $x =$ 1

3. $\sin \frac{5\pi}{3} = -\sin \frac{2\pi}{3} = -\frac{\sqrt{3}}{2}$

4. $D\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

5. $\int \sin \frac{x}{2}\, dx = -2\cos \frac{x}{2} + C$

6. $\tan(-\frac{5\pi}{6}) =$ $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

7. $\sin^{-1}(-\frac{1}{\sqrt{2}}) =$ $-\frac{\pi}{4}$

8. $D(\tan \pi) =$ 0

9. $D\sec x =$ $\sec x \tan x$

10. $\int \frac{1}{1+x^2}\, dx =$ $\tan^{-1} x + C$

11. $D\csc 5x =$ $-\csc 5x \cot 5x \cdot 5$

12. $\sin(\sin^{-1} x) =$ x

13. $\int \sec^2 \frac{x}{3}\, dx =$ $3\tan \frac{x}{3} + C$

14. $D\, 2^x = De^{x\ln 2} = \underline{e^{x\ln 2}\ln 2 = 2^x \ln 2}$

15. $\tan^{-1}(-\frac{1}{\sqrt{3}}) =$ $-\frac{\pi}{6}$

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(5) II. Find the derivative of $f(x) = \ln(\cos 2x)$.

$$f'(x) = \frac{1}{\cos 2x} \cdot -\sin 2x \cdot 2 = \frac{-2\sin 2x}{\cos 2x}$$

$$\text{or } -2\tan 2x$$

(10) III. Suppose a certain radioactive substance has a half-life of 10 days(i.e. in 10 days half of it will be left). How long would it take until only 1/3 is left?

$$f(t) = Ce^{kt} \qquad f(0) = C_0 e^0 = C_0$$

$$f(10) = \frac{C_0}{2} =$$

$$C_0 e^{k10} = \frac{C_0}{2}$$

$$e^{10k} = \frac{1}{2}$$

$$10k = \ln\frac{1}{2} = -\ln 2$$

$$k = -\frac{1}{10}\ln 2$$

$$f(t) = C_0 e^{-(\frac{1}{10}\ln 2)\cdot t} = \frac{C_0}{3} \ ?$$

$$e^{-\frac{1}{10}\ln 2\, t} = \frac{1}{3} \qquad -\frac{1}{10}\ln 2\, t = \ln\frac{1}{3} = -\ln 3$$

$$\boxed{\cancel{\tfrac{1}{10}}\, t = 10\frac{\ln 3}{\ln 2}}$$

IV. Integrate:

(10) 1. $\displaystyle\int_0^{\cancel{\pi/2}\ 3/8} \frac{1}{\sqrt{9-16x^2}}\,dx =$

$$\int_0^{\cancel{\pi/2}\ 3/8} \frac{1/3}{\sqrt{1-\left(\frac{4x}{3}\right)^2}}\,dx = \frac{1}{\cancel{3}4}\sin^{-1}\frac{4x}{3}\Big|_0^{\cancel{\pi/2}\ 3/8}$$

$$u = \frac{4x}{3}$$

$$du = \frac{1}{3}dx$$

$$\frac{1}{4}\int_{x=0}^{x=\cancel{\pi/2}} \frac{1}{\sqrt{1-u^2}}\,du = \frac{\sin^{-1}u}{4}\Big|_{x=0}^{x=\cancel{\pi/2}\ 3/8}$$

$x=0 \quad u=0$

$x = \cancel{\pi/2}\ 3/8 \quad u = \cancel{\frac{2\pi}{3}}$

$\cancel{\pi/2}$

$\sin^{-1}(\tfrac{1}{2})$

$\frac{\pi}{6}$

$$= \frac{1}{4}\sin^{-1}\cancel{\frac{2\pi}{3}}\ \tfrac{1}{2} - \frac{\sin^{-1}0}{4} \quad (\sin^{-1}0 = 0)$$

$$= \frac{1}{4}\cdot\frac{\pi}{6} = \frac{\pi}{24}$$

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IV.

2. $\displaystyle\int \frac{\cos x}{(1+3\sin x)^{5/2}}\,dx = \frac{1}{3}\,\frac{(1+3\sin x)^{-3/2}}{-3/2} + C$

$= -\frac{2}{9}(1+3\sin x)^{-3/2} + C$ or

$-\dfrac{2}{9(1+3\sin x)^{3/2}} + C$

$u = 1 + 3\sin x$

$du = 3\cos x\,dx$

$\displaystyle\frac{1}{3}\int \frac{1}{u^{5/2}}\,du = \frac{1}{3}\,\frac{u^{-3/2}}{-3/2} + C$

3. $\displaystyle\int x^3\sqrt{1-x^2}\,dx =$

① $u = 1-x^2$

$du = -2x\,dx$

$\displaystyle -\frac{1}{2}\int (u-1)\sqrt{u}\,du$

$\displaystyle = -\frac{1}{2}\int u^{3/2} - u^{1/2}\,du$

$\displaystyle = -\frac{1}{2}\,\frac{u^{5/2}}{5/2} - \frac{1}{2}\,\frac{u^{3/2}}{3/2} + C$

$= +\frac{1}{5}(1-x^2)^{5/2} - \frac{1}{3}(1-x^2)^{3/2} + C$

② $u = x^2 \quad dv = x(1-x^2)^{1/2}dx$

$du = 2x\,dx \quad v = -\frac{1}{3}(1-x^2)^{3/2}$

$\displaystyle = \frac{x^2}{3}(1-x^2)^{3/2} + \frac{1}{3}\int -2x(1-x^2)^{3/2}dx$

$= -\frac{x^2}{3}(1-x^2)^{3/2} - \frac{1}{3}\frac{2}{5}(1-x^2)^{5/2} + C$

$= -\dfrac{x^2(1-x^2)^{3/2}}{3} - \frac{2}{15}(1-x^2)^{5/2} + C$

③

$\sin u = x$

$\cos u\,du = dx$

$\sqrt{1-x^2} = \cos u$

$\displaystyle\int \sin^3 u \cos u \cos u\,du$

$\displaystyle\int \sin^3 u\cos^2 u\,du$

$\displaystyle = \int \cos^2 u(1-\cos^2 u)\sin u\,du$

$\displaystyle = \int \cos^2 - \cos^4 \sin = -\frac{\cos^3}{3}$

$-\frac{\cos^{3}u}{3} - \frac{\cos^{5}u}{5} = -\frac{(1-x^2)^{3/2}}{3} - \frac{(1-x^2)^{5/2}}{5} + C$

4. $\displaystyle\int_1^e x^2\ln x\,dx = \frac{x^3}{3}\ln x\Big|_1^e - \int_1^e \frac{x^2}{3}\,dx = \frac{x^3}{3}\ln x - \frac{x^3}{9}\Big|_1^e$ -2

$u = \ln x \qquad dv = x^2dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^3}{3}$

$= \frac{e^3}{3}\ln e - \frac{e^3}{9}$

$- \frac{1}{3}\ln 1 + \frac{1}{9}$

$= \frac{e^3}{3} - \frac{e^3}{9} - \frac{1}{3} + \frac{1}{9}$

$= \frac{2e^3}{9} + \frac{1}{9}$

or $\cancel{\frac{2}{9}(e^3-1)}\ \frac{2}{9}e^3 + \frac{1}{9}$

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