MATH 231

TEST I
January 20, 1972

NAME *Corrections*

SHOW WORK FOR FULL CREDIT!

(8) 1. Complete the following definition: A bounded function f is integrable on $[a,b]$ if and only if

$$\underline{\int_a^b} f(x)\,dx = \overline{\int_a^b} f(x)\,dx \checkmark$$

(8) 2. State precisely the Fundamental Theorem of Calculus.

If f is cont for all x in $[a,b]$, and $F'(x) = f(x)$,
then
$$\int_a^b f(x)\,dx = F(b) - F(a).$$

(8) 3. What is $\overline{\int_{-1}^{2}} f(x)dx$, where $f(x) = \begin{cases} 1, & x \text{ rational} \\ -1, & x \text{ irrational} \end{cases}$

Explain!

All Upper sums $= \sum (1)\,\Delta x_i = 3$

so $$\overline{\int_{-1}^{2}} f(x)\,dx = 3$$

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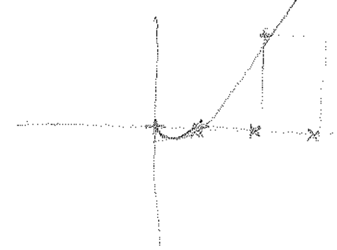
(8) 4. Find the lower sum for $f(x) = x^3 - x$ on $[0,3]$, using the partition $P = \{0,1,2,3\}$ (Set up, but do not carry out final arithmetic).

$3x^2 - 1$ $x = \pm\frac{1}{\sqrt{3}}$



$\left[\left(\frac{1}{3}\right)^3 - \frac{1}{3}\right] 1 + 0\,[1] + \left[2^3 - 2\right] \cdot 1$

-3 $\cancel{\frac{1}{27} - \frac{1}{3}}$ $+8-2$

$\cancel{\frac{3}{24}}$ $+6 = 6 - \frac{8}{24} = 5\frac{16}{24}$

$\left(\frac{1}{\sqrt{3}}\right)^3 - \frac{1}{\sqrt{3}}$

$\left(\frac{1}{3} - 1\right)\frac{1}{\sqrt{3}} = -\frac{2}{3\sqrt{3}}$ $\qquad -\frac{2}{3\sqrt{3}} + 6$

(15) 5. Evaluate the following:

a) $\lim_{n\to\infty} \frac{n-1}{2n-3} = \frac{1}{2}$ $\qquad \frac{1 - 1/n}{2 - 3/n} \begin{matrix}\to 1 \\ \to 2\end{matrix} \longrightarrow \frac{1}{2}$

b) $\sum_{k=1}^{3} (3k+1) = (3+1) + (6+1) + (9+1)$

$= 21$

c) $\lim_{n\to\infty} \frac{1}{n^2} \sum_{i=1}^{n} i = \frac{1}{n^2} \frac{n(n+1)}{2} = \frac{n+1}{2n} = \frac{1 + 1/n}{2} \longrightarrow \frac{1}{2}$

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(48) 6. Evaluate the following integrals.

a) $\int_0^2 x^2 + 1 \; dx \; = \; \frac{x^3}{3} + x \Big|_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$

b) $\int_1^2 \frac{x^2+1}{x^2} \, dx \; = \int_1^2 1 + x^{-2} dx = x + \frac{x^{-1}}{-1} \Big|_1^2$

$= 2 - \frac{1}{2} - [1 - 1] = \frac{3}{2}$

c) $\int_1^2 \frac{x}{(x^2+2)^3} \, dx \; = \frac{1}{2}\int_3^6 u^{-3} du = \frac{1}{2}\frac{u^{-2}}{-2} \Big|_3^6$

$u = x^2 + 2$

$du = 2x\,dx$

$x = 1 \quad x = 2$

$u = 3 \quad u = 6$

$= -\frac{1}{4}\left[\frac{1}{36} - \frac{1}{9}\right]$

$= -\frac{1}{4}\left[\frac{1-4}{36}\right] = \cancel{\frac{2}{36}} = \frac{1}{18}$

$\frac{3}{144}$

d) $\int_0^4 \sqrt{x}(x+1)dx \; = \int_0^4 x^{3/2} + x^{1/2} dx = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} \Big|_0^4$

$= \frac{2}{5} 4^{5/2} + \frac{2}{3} \cdot 4^{3/2} - 0$

$= \frac{64}{5} + \frac{16}{3} = \frac{192 + 80}{15}$

$= \frac{272}{15}$

$\overline{32}$

e) $\int_{-1}^{0} \frac{x}{(2x-1)}\, dx = \int_{-3}^{-1} \frac{\frac{u+1}{2}}{u} \frac{1}{2} du = \frac{1}{2}\int_{-3}^{-1} 1 + u^{-1}\, du$

$u = 2x-1$
$x = \frac{u+1}{2}$
$du = 2dx$

$= \frac{1}{2} \cancel{u\ u^{-1}}$

$x=-1$
$u=-3$
$x=0$
$u=-1$

Full credit given

Can't do at this stage

f) $\int 3x^3 - x + \frac{2}{x^2}\, dx$ — put on board as

$$\int 3x^3 - x + \frac{2}{x^2}\, dx$$

$$\frac{3x^4}{4} - \frac{x^2}{2} + \frac{2x^{-1}}{-1} \Big| = \frac{3x^4}{4} - \frac{x^2}{2} - \frac{2}{x} + C$$

-3

(5) 7. True or false

$$\int \frac{x}{\sqrt{x-1}}\, dx = 2\sqrt{x-1} + c$$

Explain.

$$\int \frac{x}{\sqrt{x-1}}\, dx = 2\sqrt{x-1} + c$$

False $D\, 2\sqrt{x-1} = (x-1)^{-1/2} = \frac{1}{\sqrt{x-1}}$ not $\frac{x}{\sqrt{x-1}}$

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ASSIGNMENT: Start reading Chapter 10.