Math 231
Test II
Feb. 8, 1974

Name KEY

Show work!

(10) 1. Complete the following definitions:

a. $\ln x = \int_1^x \frac{1}{t}\,dt,\ x > 0$

b. $y = e^x$ iff $x = \ln y$

(24) 2. Quick answer. No partial credit.

a. If $y = e^7$, then $\ln y = 7$

b. $\ln e = 1$

c. If $f(x) = e^2$, then $f'(x) = 0$

~~d. If $y = 2^5$, then $\ln y = 5 \ln 2$~~

$2^5 = e^{5 \ln 2}$

e. $\int e^{-5x}dx = \frac{e^{-5x}}{-5} + C$

f. If $\log_{10} x = 0$, then $x = 1$

g. $\log_3 x = 3$, then $x = 3^3 = 27$

h. $\ln(x^2 - 1) = 1$, then $x = \pm\sqrt{1+e}$

$x^2 - 1 = e$

$x^2 = 1 + e$

34

(10) 3. <u>Write out</u> the approximation given by Simpson's Rule for the integral $\int_3^6 x^2-4\, dx$. Use 6 subintervals. Do <u>not</u> do arithmetic.

X	Y
3	$3^2-4 = 5$
$3\frac{1}{2}$	$(\frac{7}{2})^2-4 = \frac{49}{4}-4 = \frac{33}{4}$
4	$4^2-4 = 12$
$4\frac{1}{2}$	$(\frac{9}{2})^2-4 = \frac{81}{4}-\frac{16}{4} = \frac{65}{4}$
5	$25-4 = 21$
$5\frac{1}{2}$	$(\frac{11}{2})^2-4 = \frac{121}{4}-\frac{16}{4} = \frac{105}{4}$
6	$36-4 = 32$

$$\frac{6-3}{3\cdot 6}\left[5+4\left(\frac{33}{4}\right)+2(12)+4\left(\frac{65}{4}\right)+2(21)+4\left(\frac{105}{4}\right)+32\right]$$

$$\frac{1}{6}\left[5+33+24+65+42+105+32\right]$$

$$\frac{306}{6} = 51$$

$$\begin{array}{r} 32 \\ 105 \\ 42 \\ 65 \\ 24 \\ 33 \\ 5 \\ \hline 306 \end{array}$$

$$\int_3^6 x^2-4\,dx = \frac{x^3}{3}-4x\Big|_3^6 = \frac{36\cdot\cancel{6}^2}{\cancel{3}}-24-[9-12] = 72-24-9+12 = 51$$

(20) 4. Find the following derivatives:

a. $D(\ln x + e^x - 2^x + x\ln 3) =$ $e^{x\ln 2}$

$$\frac{1}{x}+e^x-2^x\ln 2+\ln 3$$

b. $D(\ln(x^3-3)) =$ $\frac{1}{x^3-3}\cdot 3x^2$

c. $f(x) = 2e^{x^2}$, $f''(x) =$

$$f'(x) = 2e^{x^2}\cdot 2x = 4xe^{x^2}$$

$$f''(x) = 4x(e^{x^2}\cdot 2x)+4e^{x^2}$$

$$= (8x^2+4)e^{x^2}$$

d. $D\sqrt{1-e^x} = D(1-e^x)^{1/2}$

$$\frac{1}{2}(1-e^x)^{-1/2}(-e^x)$$

$$= \frac{-e^x}{2\sqrt{1-e^x}}$$

30

5. Compute each of the following integrals:

9 a. $\int e^{2x} - \frac{1}{2x-1} + e^{x-1}\, dx = \frac{e^{2x}}{2} - \frac{1}{2}\ln|2x-1| + e^{x-1} + C$

9 b. $\int_1^3 \frac{(\ln x)^3}{2x}\, dx = \frac{1}{2}\int_{x=1}^{3} u^3\, du = \frac{1}{2}\frac{u^4}{4}\Big|_{x=1}^{3}$ or

$u = \ln x$
$du = \frac{1}{x}\, dx$

$= \frac{1}{8}(\ln x)^4\Big|_1^3 = \frac{1}{8}(\ln 3)^4 - \frac{1}{8}(\underset{0}{\ln 1})^4$

$= \frac{1}{8}(\ln 3)^4$

9 c. $\frac{1}{3}\int_1^3 3x^2\, e^{x^3-5}\, dx = \frac{1}{3}\, e^{x^3-5}\Big|_1^3$

$= \frac{1}{3}(e^{22} - e^{-4})$

27

5. cont.

9 d. $\int \frac{3x^2}{x-10}\,dx = 3\int \frac{(u+10)^2}{u}\,du = 3\int \frac{u^2+20u+100}{u}\,du$

$= 3\int u + 20 + \frac{100}{u}\,du$

(1) $u = x-10$

$du = dx$

$= 3\left[\frac{u^3}{2} + 20u + 100\ln|u|\right] + C$

$x = u+10$

$x^2 = (u+10)^2$

$= \frac{3}{2}\left[\frac{(x-10)^2}{2} + 20(x-10) + 100\ln|x-10|\right] + C$

$= \frac{3}{2}(x-10)^2 + 60(x-10) + \cancel{150}\,300\ln|x-10| + C$

(2)

$$\begin{array}{r} 3x+30 \\ x-10\,\overline{)\,3x^2 \qquad} \\ \underline{3x^2-30x} \\ 30x \qquad \\ \underline{30x-300} \\ 300 \end{array}$$

$\int 3x + 30 + \frac{300}{x-10}\,dx$

$\frac{3x^2}{2} + 30x + 300\ln|x-10| + C$