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10 in

Math 231
Test III
March 1, 1974

Name KEY

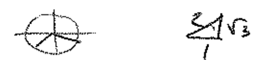
Show work for partail credit!

(30) I. Short answer. No partial credit.

1. If $f'(x) = k \cdot f(x)$, then $f(x) =$ Ce^{kx}

2. $D(\cos \pi) =$ 0

3. If $\sin^{-1} y = \frac{\pi}{4}$, then $y =$ $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$



4. $\tan \frac{5\pi}{4} =$ 1

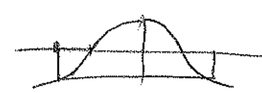
5. $\sin(-\frac{\pi}{6}) =$ $-\frac{1}{2}$

6. $D \tan x =$ $\sec^2 x$

7. $D \tan^{-1} x =$ $\frac{1}{1+x^2}$

8. $\int \cos 2x \, dx =$ $\frac{\sin 2x}{2} + c$

9. $\tan^{-1}(-\sqrt{3}) =$ $-\frac{\pi}{3}$



10. $\cos^{-1}(\cos(-\frac{3\pi}{4})) =$ $\frac{3\pi}{4}$

(10) II. Suppose that the bacteria count of a certain culture growing under ideal conditions is initially 1500, and after 24 hours the count is 2000. What will be the count after 30 hours?

$f(t) = Ce^{kt}$

$1500 = Ce^{k0}$

$c = 1500$

$2000 = Ce^{24k}$

$\frac{4}{3} = e^{24k}$

$\ln \frac{4}{3} = 24k$

$k = \frac{1}{24} \ln \frac{4}{3}$

$f(t) = 1500\, e^{\frac{1}{24} \ln(\frac{4}{3}) t}$

$f(30) = 1500\, e^{\frac{1}{24} \ln(\frac{4}{3}) \cdot 30}$

$= 1500\, e^{\frac{5}{4} \ln \frac{4}{3}}$

or $= 1500 \left(\frac{4}{3}\right)^{5/4}$

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(20) III. Find the following derivatives:

1. $D(\sin 2x)^3 =$ $3(\sin 2x)^2 \cos 2x \cdot 2 = 6(\sin 2x)^2 \cos 2x$

2. $D \tan(x^3-1) =$ $\sec^2(x^3-1)(3x^2) = 3x^2 \sec^3(x^3-1)$

3. $D \sin^{-1}(x^2) =$ $\dfrac{2x}{\sqrt{1-x^4}}$

4. $D(\sec x \tan x)$ $= \sec x \sec^2 x + \tan x \sec x \tan x$

$= \sec^3 x + \sec x \tan^2 x$

$= \sec(\sec^2 + \tan^2 x)$

(40) IV. Find the following integrals:

(8) 1. $\displaystyle\int_0^2 \frac{x-1}{x^2+4}\,dx =$ (Hint: Split into the sum of 2 fractions.)

$\displaystyle\int_0^2 \frac{x}{x^2+4}\,dx - \int_0^2 \frac{1/4}{\frac{x^2}{4}+1}\,dx$

$= \frac{1}{2}\ln(x^2+4)\Big|_0^2 - \frac{1}{2}\tan^{-1}\frac{x}{2}\Big|_0^2$

$= \frac{1}{2}\ln(8) - \frac{1}{2}\ln 4 - \frac{1}{2}\tan^{-1}1 + \frac{1}{2}\tan^{-1}0$

$= \frac{1}{2}\ln 2 - \frac{\pi}{8} + 0$

(8) 2. $\displaystyle\int 2x\, e^{3x}\,dx =$ $\frac{2x}{3}e^{3x} - \int \frac{2}{3}e^{3x}\,dx$

$u = 2x \quad dv = e^{3x}\,dx$

$= \frac{2}{3}xe^{3x} - \frac{2}{9}e^{3x} + c$

$du = 2\,dx \quad v = \frac{e^{3x}}{3}$

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3. $\int \cos^3 x \, dx = \int \cos x (1-\sin^2 x)\, dx = \int 1-u^2 \, du$

$u = \sin x$
$du = \cos x \, dx$

$= u - \frac{u^3}{3} + C$

$= \sin x - \frac{\sin^3 x}{3} + C$

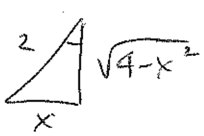
(8) 4. $\int \frac{\sec^2 x}{1+\tan x} dx$ $\quad \int \frac{1}{u} du = \ln|u| + C$

$u = 1 + \tan x$
$du = \sec^2 x \, dx$

$= \ln|1+\tan x| + C$

(8) 5. $\int_0^2 \sqrt{4-x^2}\, dx =$ (Try a trigonometric substitution.)

$\int_0^{\pi/2} 2\cos u \, 2\cos u \, du = 4\int_0^{\pi/2} \cos^2 u \, du$



$\sin u = \frac{x}{2}$

$\cos u \, du = \frac{1}{2} dx$

$\frac{\sqrt{4-x^2}}{2} = \cos u$

$= 4\int_0^{\pi/2} \left(\frac{1+\cos 2u}{2}\right) du$

$= 2u + \sin 2u \Big|_0^{\pi/2}$

$= \pi + \sin \pi - 0 - \sin 0$

$= \pi$