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Math 231
Test I
Jan. 24, 1975

Name KEY

Show work for full credit!

I. Short answer: 5 points ea.

a. $\int 4x^3 + 3x\,dx = \frac{4x^4}{4} + 3\frac{x^2}{2} + c = x^4 + \frac{3}{2}x^2 + C$

b. $\int_1^2 x^2 + 2\sqrt{x}\,dx = \frac{x^3}{3} + \frac{2x^{3/2}}{3/2}\Big|_1^2 = \frac{x^3}{3} + \frac{4}{3}x^{3/2}\Big|_1^2 = \frac{8}{3} + \frac{4(2\sqrt{2})}{3} - \frac{1}{3} - \frac{4}{3}$
$= 1 + \frac{8\sqrt{2}}{3}$

c. Write out: $\sum_{i=1}^{5} \frac{1}{i+1} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

d. $\int x^7 + \frac{3}{x^3}\,dx = \frac{x^8}{8} + \frac{3x^{-2}}{-2} + c = \frac{x^8}{8} - \frac{3}{2x^2} + c$

e. $\int \frac{1}{(2x+1)^6}\,dx = \frac{1}{2}\int 2(2x+1)^{-6}dx = \frac{1}{2}\frac{(2x+1)^{-5}}{-5} + c = -\frac{(2x+1)^{-5}}{10} + c$

f. $\frac{1}{3}\int (x^3-3)^5 3x^2\,dx = \frac{1}{3}\frac{(x^3-3)^6}{6} + c = $
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g. If $f(x) = \int_1^x \sqrt{1+t^2}\,dt$, then $f'(x) = \sqrt{1+x^2}$

h. The area of the region bounded by the curves $y = x^3$, $y = 0$, $x = 0$, and $x = 2$ is

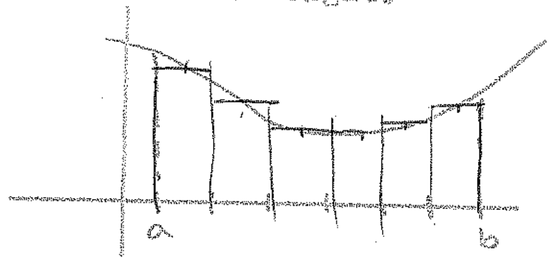
$$\int_0^2 x^3 dx = \frac{x^4}{4}\Big|_0^2 = \frac{16}{4} - 0 = 4$$

i. In order to apply the fundamental theorem of calculus to evaluate the definite integral

$\int_a^b f(x)\,dx$, we must know that... cont for $a \le x \le b$

Q

10. For the following graph draw the approximating rectangles ~~under~~ for the curve for the given subdivision, using the value at the midpoints of each subinterval as the height.



II. Problems. 10 points each.

a. Evaluate $\int_0^1 3x^2\,dx$ using the definition.

Divide the interval into n parts, write out the subdivision points, compute S_n for the circumscribed area, and find its limit. Draw the graph with n = 4 rectangles.

$\Delta x = \frac{1}{n}$

$0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots, \frac{n}{n} = 1$

$$S_n = 3\left(\tfrac{1}{n}\right)^2\cdot\tfrac{1}{n} + 3\left(\tfrac{2}{n}\right)^2\cdot\tfrac{1}{n} + \dots + 3\left(\tfrac{n}{n}\right)^2\cdot\tfrac{1}{n}$$

$$= \frac{3}{n^3}\left[1^2+2^2+\dots+n^2\right] = \frac{3}{n^3}\;\frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1}{2}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) = \frac{1}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$\lim_{n\to\infty} S_n = \frac{1}{2}\cdot 1\cdot 2 = 1$$

ck $\int 3x^2 = x^3\Big|_0^1 =$

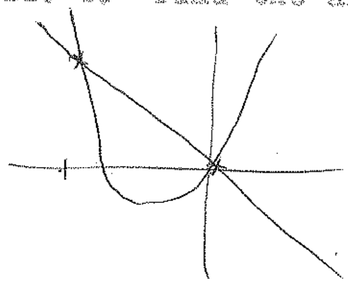
Recall:

$1+2+\dots+n = \frac{n(n+1)}{2}$

$1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$1^3+2^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

II. b. Find the area between the curves $y = x^2+2x$ and $y = -x$.



$$-x = x^2+2x$$

$$x^2+3x=0$$

$$x=0 \quad x=-3$$

$$\int_{-3}^{0} -x-(x^2+2x)\,dx = \int_{-3}^{0} -x^2-3x\,dx$$

$$= -\frac{x^3}{3} - \frac{3x^2}{2}\Big|_{-3}^{0} = 0-\left(-\frac{(-3)^3}{3} - \frac{3(-3)^2}{2}\right)$$

$$= -\frac{27}{3} + \frac{27}{2} = \frac{27}{6} = \frac{9}{2}$$

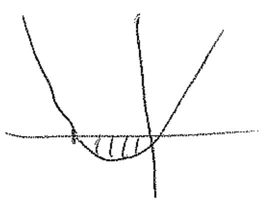
c. Use the trapezoidal rule to approximate $\int_0^2 \frac{1}{1+x^2}\,dx$. Use 5 subintervals. Set up <u>only</u>, do not do arithmetic.

0 .4 .8 1.2 1.6 2

$$\Delta x = \frac{2}{5} = .4$$

$$T_5 = \frac{2-0}{5}\left[\frac{1}{2}\frac{1}{1+0^2} + \frac{1}{1+(.4)^2} + \frac{1}{1+(.8)^2} + \frac{1}{1+(1.2)^2} + \frac{1}{1+(1.6)^2} + \frac{1}{2}\,\frac{1}{1+2^2}\right]$$

d. Find the volume of the solid generated by revolving about the x-axis the region bounded by $y = x^2 + x$ and the x-axis.



$$x(x+1)$$

$$A(x) = \pi(x^2+x)^2$$

$$V = \int_{-1}^{0} \pi(x^2+x)^2\,dx = \pi\int_{-1}^{0} x^4+2x^3+x^2\,dx$$

$$= \pi\left[\frac{x^5}{5} + \frac{2x^4}{4} + \frac{x^3}{3}\right|_{-1}^{0} = \pi\left[0-\left(\frac{-1}{5}+\frac{1}{2}-\frac{1}{3}\right)\right)$$

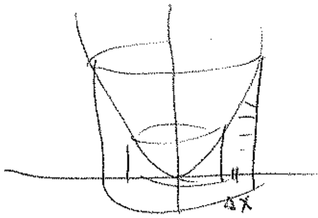
$$= \pi\left(\frac{1}{5}-\frac{1}{2}+\frac{1}{3}\right)$$

$$= \pi\,\frac{6-15+10}{30} = \frac{\pi}{30}$$

II. e. Do ONE of the following:

1. Find the volume of the solid generated by revolving about the y-axis the region bounded by $y = x^2$, $x = 1$, $x = 2$, $y = 0$.

2. Find the volume of the solid whose base is the region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$, $x = 1$, and whose cros~~s~~sections perpendicular to the ~~x~~-axis are squares.

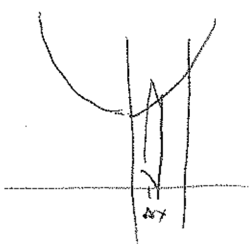
1. 

shells $2\pi x\,(x^2)\,dx$

$$V = \int_1^2 2\pi x^3\,dx = \frac{2\pi x^4}{4}\Big|_1^2$$

$$= \frac{32\pi}{4} - \frac{2\pi}{4}$$

$$= \frac{30}{4}\pi = \frac{15}{2}\pi$$

2. 

$$A(x) = (x^2+1)^2 = x^4 + 2x^2 + 1$$

$$V = \int_0^1 x^4 + 2x^2 + 1\,dx$$

$$= \frac{x^5}{5} + \frac{2x^3}{3} + x\Big|_0^1$$

$$= \frac{1}{5} + \frac{2}{3} + 1 - 0 = \frac{3 + 10 + 15}{15} = \frac{28}{15}$$