Math 251
Test II
Feb. 14, 1975

11102
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Name KEY

Show work!

I. 4 points each.

1. $y = e^x$ if and only if $x = \ln y$

2. DEF $y = \tan^{-1} x$ if and only if $x = \tan y$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

3. DEF $\ln x = \int_1^x \frac{1}{t}\,dt$

4. $\frac{d}{dx} \sec 2x = \sec 2x \tan 2x \cdot 2$

5. $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

6. $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$

7. $\frac{d}{dx} \tan^{-1} \frac{x}{2} = \frac{1}{1+(\frac{x}{2})^2} \cdot \frac{1}{2} = \frac{1}{1+\frac{x^2}{4}} \cdot \frac{1}{2} = \frac{2}{4+x^2}$

8. $\frac{d}{dx} \ln 3 = 0$

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9. $\frac{d}{dx}(1+\tan x)^3 = 3(1+\tan x)^2 \sec^2 x$

10. $e^{2\ln 5} = 5^2 = 25$

11. $\int_1^e \frac{2}{x}\,dx = 2\ln e = 2$

12. $\int 5\cos 3x\,dx = \frac{5}{3}\sin 3x + C$

13. $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x\Big|_0^{1/2} = \sin^{-1}\frac{1}{2} - \sin^{-1}0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$

14. $\int e^{3x+1}\,dx = \frac{e^{3x+1}}{3} + C$

15. $\frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1) + C$

16. $\frac{d}{dx}x^\pi = \pi x^{\pi-1}$

17. $\ln(\ln e) = \ln 1 = 0$

18. $\int \frac{\sec^2 x}{\tan x}\,dx =$ $\ln|\tan x| + C$

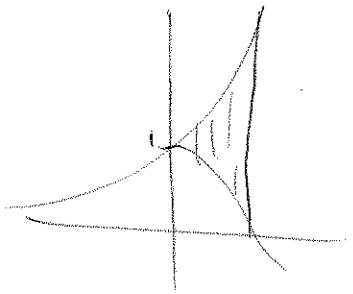
19. $\int \frac{3}{5x+8}\,dx =$ $\frac{3}{5}\ln|5x+8| + C$

20. If $\frac{dy}{dx} = 3y$, and $y=2$ when $x=0$, then $y =$ $2e^{3x}$

$y = Ce^{3x}$

II. 10 points each.

1. Find the area of the region bounded by the curves $y = e^x$, $y = \cos x$, $x = \frac{\pi}{2}$, for $x > 0$.



$$\int_0^{\pi/2} e^x - \cos x\,dx = e^x - \sin x \Big|_0^{\pi/2}$$

$$= e^{\pi/2} - \sin \pi/2 - (e^0 - \sin 0)$$

$$= e^{\pi/2} - 1 - 1 + 0$$

$$= e^{\pi/2} - 2$$

2. A bacteria culture has a count of 1000 which increases to a count of 2500 in 1 hour. If the rate of growth is proportional to the amount, what will be the count at the end of 3 hours (from the start.)

$y = 1000 \quad t = 0$

$y = 2500 \quad t = 1$

$y = ? \quad t = 3$

$y = Ce^{kt}$

$C = 1000$

$y = 1000e^{kt}$

$2500 = 1000e^{k1}$

$e^k = 2.5$

$k = \ln(2.5)$

$y = 1000e^{\ln 2.5\, t}$

$t = 3 \quad y = 1000e^{\ln(2.5)3}$

$= 1000(2.5)^3$

$= 15625$

2.5
2.5
6.25
2.5
3125
1250
15.625