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Math 231
Final Exam
March 17, 1976

Name KEY

12 min
Time OK for 2 hrs
First to leave
1½ hrs

I. Short Answer!

a. $\int 3\, e^{-2x}\, dx = \dfrac{3e^{-2x}}{-2} + C$

b. The trapezoidal rule approximates the area under a curve with the sum of areas of trapezoids. Cpmplete: Simpson's rule approximates this area with ... areas of pieces of parabolas

med = 120

c. If $\frac{dy}{dx} = 3y$, then $y = Ce^{3x}$

d. Write out the partial fractions expansion (Do NOT solve!) for

$$\frac{x^2+1}{x(x-1)^2(x^2+4)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} + \frac{Dx+E}{(x^2+4)} + \frac{Fx+G}{(x^2+4)^2}$$

e. $\int \sin(4x+1)\, dx = -\dfrac{\cos(4x+1)}{4} + C$

f. $\int \dfrac{x}{x^2+1}\, dx = \frac{1}{2} \ln(x^2+1) + C$

g. If $y = \csc(e^x)$, then $\frac{dy}{dx} = -\csc e^x \cot e^x \cdot e^x$

h. $\frac{d}{dx}(\sec^2(2x)) = 2 \sec(2x) \sec 2x \tan 2x \cdot 2$
$= 4 \sec^2(2x) \tan(2x)$

i. $\frac{d}{dx}(e^{\sin x}) = e^{\sin x} \cos x$

j. $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

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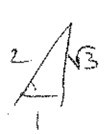

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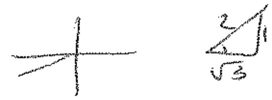
I. cont.

k. A. $\int_1^2 \frac{1}{x-1}\,dx$ B. $\int_1^\infty \frac{1}{x-1}\,dx$

(✓) Both are improper integrals. ( ) A is improper and B is not. ( ) B is improper and A is not. ( ) both are proper integrals.

l. $\sin(\frac{7\pi}{6}) = -\frac{1}{2}$



m. DEF: $y = \sin^{-1}x$ if and only if $x = \sin y$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

n. DEF: $y = e^x$ if and only if $x = \ln y$

o. Simplify: $e^{2\ln 6} = 6^2 = 36$

p. $\frac{d}{dx} \sin \pi = 0$

q. $x^{1/2}$

$$\int \frac{x^5}{2} - \frac{2x^2}{\sqrt{x}}\,dx = \frac{x^6}{12} - \frac{\cancel{2}x^{5/2}}{5/\cancel{2}} + C \qquad \frac{x^6}{12} - \frac{4}{5}x^{5/2} + c$$

r. Find the area of one "piece" of the region bounded by the curve $y = \sin x$ and the x-axis.

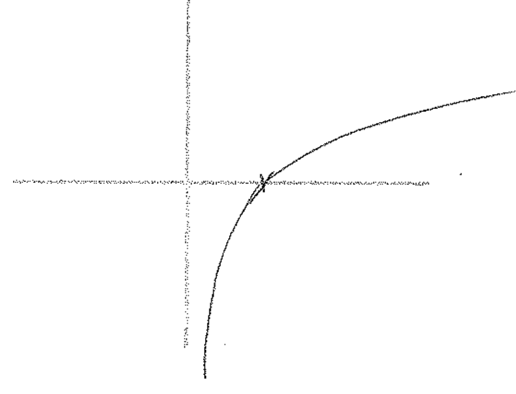
$$\int_0^\pi \sin x\,dx = -\cos x\Big|_0^\pi = -\cos\pi + \cos 0 = -(-1) + 1 = 2$$

s. The arc length of the curve $y = e^x$ between (0,1) and (1,e) is (Set up) $y' = e^x$

$$\int_0^1 \sqrt{1 + e^{2x}}\,dx$$

BONUS: 15 pts, integrate it! (Do on back.) Circle this if done.

t. Sketch $y = \ln x$ carefully:



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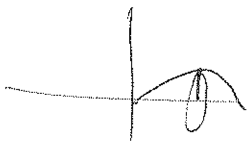
120 10 pts 12

II. a. Find the volume of one "piece" of the solid generated by revolving the region bounded by y = sinx and the x-axis about the the x-axis.

BONUS Find the volume if the region is revolved about the y axis.



$$\int_0^{\pi} \pi \sin^2 x\, dx = \int_0^{\pi} \pi \, \frac{1-\cos 2x}{2}\, dx \quad 5$$

$$= \pi\left[\frac{x}{2} - \frac{\sin 2x}{4}\right]\Big|_0^{\pi} = \pi\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4}\right] - \pi\left[0\right]$$

$$= \frac{\pi^2}{2}$$

BONUS

$$\int_0^{\pi} 2\pi x \sin x\, dx$$

$$2\pi\left[-x\cos x + \sin x\right]\Big|_0^{\pi}$$

$$= 2\pi\left[-\pi(-1) + 0\right] - 2\pi\left[0\right] = 2\pi^2$$

b. If $\frac{dy}{dx} = -xy$ and y = 1 when x = 0, then y = $e^{-x^2/2}$

$$\frac{1}{y}\, dy = -x\, dx$$

$$\ln y - \cancel{3}$$

$$\ln y = \cancel{e^{-x}} -\frac{x^2}{2} + c$$

$$c = 0$$

$$y = e^{-x^2/2}$$

III. Do FIVE 10 pts

a. $\int x \sin x\, dx = -x\cos x - \int -\cos x\, dx$

$u = x \qquad dv = \sin x\, dx$

$du = dx \qquad v = -\cos x + c$

$$= -x\cos x + \sin x + c$$

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b. $\displaystyle\int \frac{1}{(x+2)(x-3)^2}\,dx = \frac{1}{25}\int \frac{1}{x+2}\,dx \mp \frac{1}{25}\int \frac{1}{x-3}\,dx + \frac{1}{5}\int \frac{1}{(x-3)^2}\,dx$

$$\frac{A}{x+2} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}$$

$$= \frac{1}{25}\ln|x+2| \mp \frac{1}{25}\ln|x-3| + \frac{1}{5}(x-3)^{-1} + C$$

$$1 = A(x-3)^2 + B(x-3)(x+2) + C(x+2)$$

$x = 3$

$1 = 0 + 0 + 5C \qquad C = \frac{1}{5}$

$x = -2$

$1 = 25A + 0 + 0$

$A = \frac{1}{25}$

$x = 0$

$1 = 9A - 6B + 2C$

$1 = \frac{9}{25} - 6B + \frac{2}{5}$

$6B = \frac{-25+9+10}{25} = \frac{6}{25} \qquad B = -\frac{1}{25}$ 16

c. $\displaystyle\int_0^\infty \frac{2}{(x+1)^2}\,dx = 2$

$$\int_0^L 2(x+1)^{-2}\,dx = -2(x+1)^{-1}\Big|_0^L = -\frac{2}{(L+1)} + 2 \longrightarrow 2$$

d. $\displaystyle\int_0^{\pi/4} \sec^4 x\,dx = \int_0^{\pi/4} (1+\tan^2 x)\sec^2 x\,dx$

$$= \tan x + \frac{\tan^3 x}{3}\Big|_0^{\pi/4} = \tan\frac{\pi}{4} + \frac{\tan^3\frac{\pi}{4}}{3} - 0$$

$$= 1 + \frac{1}{3} = \frac{4}{3}$$

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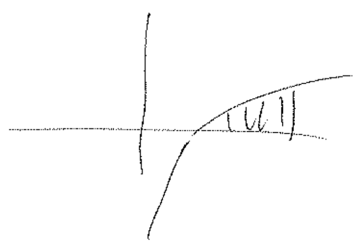
e. If $y = e^{2x}\cos 3x + (\ln \sin x)^2$, then $\frac{dy}{dx} =$

$$e^{2x}(-3\sin 3x) + 2e^{2x}\cos 3x + 2(\ln\sin x)\frac{1}{\sin x}(\cos x)$$

-4

f. The area of the region bounded by the curves $y = \ln x$, $y = 0$, and $x = 4$ is...



$$\int_{\emptyset}^{4} \ln x\, dx = x\ln x\Big|_{\emptyset}^{4} - \int_{\emptyset}^{4} 1\, dx$$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$= 4\ln 4 - 0 - 3$

$= 4(\ln 4) - 3$

18½

g.. Suppose the half life of a radioactive isotope is 10 years. How long will it take for the amount to be one tenth of the original amount? Assume the amount remaining decreases at a rate proportional to the amount present.

$P = P_0 e^{kt}$

$\frac{P_0}{2} = P_0 e^{k10}$

$\frac{1}{2} = e^{10k}$

$10k = \ln \frac{1}{2}$

$k = \frac{1}{10}\ln(\frac{1}{2})$

$P = P_0 e^{\frac{1}{10}\ln(\frac{1}{2})t}$

$\frac{1}{10} = e^{\frac{1}{10}\ln(\frac{1}{2})t}$

$\ln\frac{1}{10} = \frac{1}{10}\ln(\frac{1}{2})t$

$t = \frac{10\ln \frac{1}{10}}{\ln \frac{1}{2}} = \frac{10\ln 10}{\ln 2}$