Math 231
Test I
January 23, 1976

Name KEY

Show work for full credit!

I. Short Answer: 5 points ea.

1. $\int_1^4 x^3 + 2x\sqrt{x}\ dx = \frac{x^4}{4} + 2\frac{x^{5/2}}{5/2}\Big|_1^4 = \frac{4^4}{4} + \frac{4}{5}4^{5/2} - \left(\frac{1}{4} + \frac{4}{5}\right)$

$= 64 + \frac{128}{5} - \frac{1}{4} - \frac{4}{5} = \frac{255}{4} + \frac{124}{5}$

$\frac{1275 + 496}{20}$

$\frac{1771}{20}$

2. If the interval $2 \le x \le 5$ is partitioned into 15 equal subintervals, then $\Delta x = \frac{1}{5}$

$\frac{3}{15}$

3. $\int \frac{1}{(3x+4)^3} dx = \int (3x+4)^{-3} dx = \frac{(3x+4)^{-2}}{-2\cdot 3} + c = -\frac{1}{6(3x+4)^2} + c$

4. $\int_0^{\pi/4} \sin 2x\ dx = -\frac{\cos 2x}{2}\Big|_0^{\pi/4} = -\frac{\cos\frac{\pi}{2}}{2} + \frac{\cos 0}{2} = \frac{1}{2}$

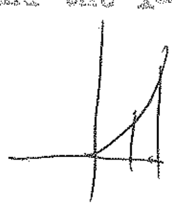
5. In order to apply the fundamental theorem of calculus to evaluate the definite integral $\int_a^b f(x)\,dx$ we must know that f is: cont.

6. $\int_2^5 x^2\left(\sqrt{x^3-4}\right)dx = \frac{1}{3}\frac{(x^3-4)^{3/2}}{3/2}\Big|_2^5 = \frac{2}{9}(x^3-4)^{3/2}\Big|_2^5$

$= \frac{2}{9}(121)^{3/2} - \frac{2}{9}4^{3/2} = \frac{2662}{9} - \frac{16}{9} = \frac{2646}{9}$

$= 294$

7. The area of the region bounded by the curves $y = 2x^3$, $x = 1$, $x = 2$ and the x-axis is:



$A = \int_1^2 2x^3\,dx = \frac{2x^4}{4}\Big|_1^2 = \frac{32}{4} - \frac{2}{4} = 8 - \frac{1}{2} = 7\frac{1}{2}$

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M = 74

II. 1. The curve $y = x^2 + 1$ is graphed below.

(10) a. Divide the interval $0 \le x \le 2$ into 5 equal subintervals on the graph. $\Delta x = \frac{2}{5} = .4$

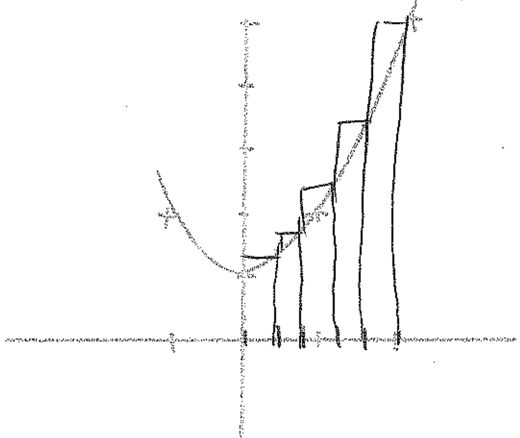
b. The points of the partition are

$x_0 = 0$, $x_1 = .4$, $x_2 = .8$, $x_3 = 1.2$, $x_4 = 1.6$,

$x_5 = 2$.

c. Draw the rectangles for the Riemann sum S_n using the right endpoint in each interval (circumscribed area) and

d. Write out (do NOT do arithmetic) this sum:



$$S_n = (.4^2+1).4 + (.8^2+1).4 + (1.2^2+1).4 + (1.6^2+1).1 + (2^2+1).1$$

(15) 2. Let $f(x) = 2x^2$. Compute the Riemann sum S_n for f on the interval $0 \le x \le 3$ using the right hand end point of each subinterval (circumscribed area). Find $\lim_{n \to \infty} S_n$. Check. What is this limit called?

$[0, 3] \qquad \Delta x = \frac{3}{n} \qquad x_i = \frac{3i}{n}$

$$S_n = \sum_{i=1}^{n} 2(x_i^2)\Delta x = \sum_{i=1}^{n} 2\left(\frac{3i}{n}\right)^2 \frac{3}{n} = \frac{2 \cdot 3^3}{n^3} \sum_{i=1}^{n} i^2$$

$$= \frac{54}{\cancel{n^3}_{\,n^2}} \frac{\cancel{n}(n+1)(2n+1)}{6}$$

$$= 9\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$= 9\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$\lim_{n \to \infty} S_n = 9 \cdot 2 = 18$$

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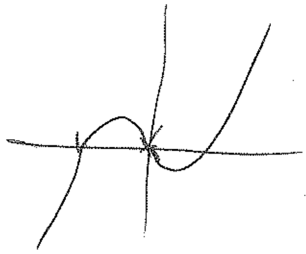
$$\int_0^3 2x^2\,dx = \left.\frac{2x^3}{3}\right|_0^3 = \frac{2 \cdot 3^3}{3} = 2 \cdot 3^2 = 18$$

III. 10 points each.

1. Find the area of the region between the curve $y = x^3 - x$ and the x axis.

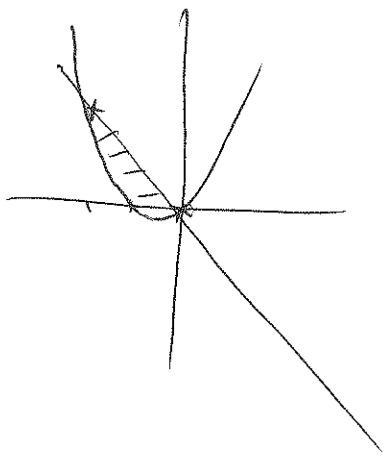


$x(x^2-1)$

$= \int_{-1}^{0} x^3 - x\,dx + \int_{0}^{1} -(x^3 - x)\,dx$

$= 2\int_{-1}^{0} x^3 - x\,dx = \frac{\cancel{2}x^4}{\cancel{4}_{2}} - \frac{\cancel{2}x^2}{\cancel{2}}\Big|_{-1}^{0} = -\left(\frac{1}{2} - 1\right) = \frac{1}{2}$

2. Find the area between the curves $y = -x$ and $y = x^2 + x$.



$-x = x^2 + x$ $\quad -1$

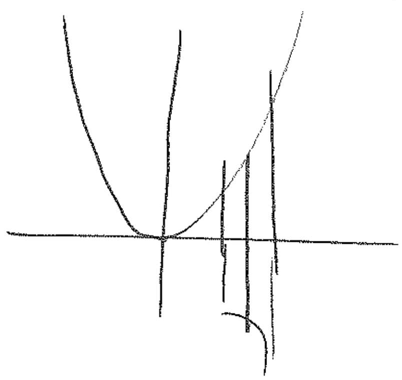
$x^2 + 2x = 0$ $\quad 1$

$x(x+2) = 0$

$\int_{-2}^{0} -x - (x^2 + x)\,dx = \int_{-2}^{0} -x^2 - 2x\,dx = -\frac{x^3}{3} - x^2\Big|_{-2}^{0}$

$= \frac{(-2)^3}{3} + (-2)^2 = -\frac{8}{3} + 4 = \frac{4}{3}$

3. The region bounded by the curves $y = 3x^2$, $x = 1$, $x = 2$ and the x axis is revolved about the x-axis to form a solid. What is its volume?



$A(x) = \pi(3x^2)^2 = 9\pi x^4$

$V = \int_{1}^{2} 9\pi x^4\,dx = \frac{9\pi x^5}{5}\Big|_{1}^{2}$

$= \frac{9\pi\, 32}{5} - \frac{9\pi}{5}$

$= \frac{9\pi}{5}(31) = \frac{279}{5}\pi$

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III. cont.

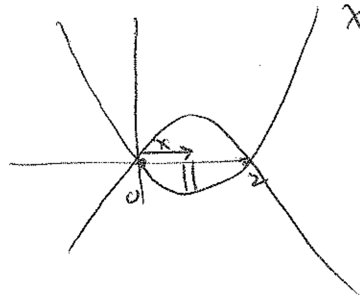
4. Do ONE.

a. The region bounded by the curves $y = x^2 - 2x$ and the x-axis is revolved about the y-axis. Find its volume.

b. The base of a solid is the region between $y = x^2$ and $y = 9$, and the cross sections perpendicular to the x-axis are triangles with height 2. Find the volume of this solid.

c. The region bounded by the curves $y^2 = x - 2$, and $x = 1$ is revolved about the line $y = 3$. Find the volume of the solid generated.

a.



$x(x-2)$

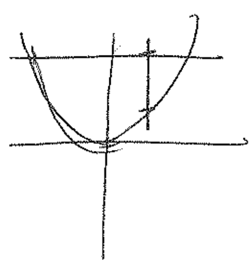
$$\int_0^2 2\pi x\, y\, dx = \int_0^2 2\pi x\,(+x^2 \mp 2x)\,dx$$

$$= 2\pi \int_0^2 -x^3 + 2x^2\,dx = 2\pi\left(-\frac{x^4}{4} + \frac{2x^3}{3}\right)\Big|_0^2$$

$$= 2\pi\left(-4 + \frac{16}{3}\right)$$

$$= \frac{8\pi}{3}$$

b.



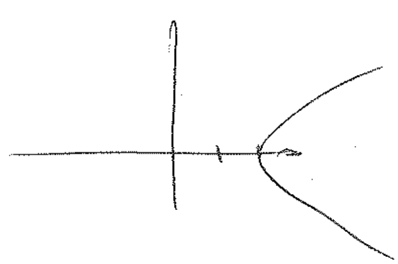
$$A(x) = \frac{(9-x^2)\,2}{2}$$

$$= 18 - 2x^2$$

$$V = \int_{-3}^{3} 18 - 2x^2\,dx = 18x - \frac{2x^3}{3}\Big|_{-3}^{3}$$

$$= 2(54 - 18)\ 2(36) = \frac{72}{2} = 36$$

c.



a $\int_0^2 2\pi x\,(-x^2+2x)\,dx$

$\int_0^2 -2\pi x^3 + 4x^2\pi\,dx$

$-\cancel{16\pi + 16\pi}$

$-\frac{2\pi x^4}{4} + \frac{4\pi x^3}{3}\Big|_0^2$

$-8\pi + \frac{32}{3}\pi = \frac{8}{3}\pi$

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