

9.05  
9.10  
5 min

Math 231  
Test II  
Feb. 13, 1976

Name KEY

short.

lots of people  
left with  
5 min to go

med = 86

✓

Show work!

I. 4 points ea.

1. Define:  $\ln x = \int_1^x \frac{1}{t} dt, x > 0$

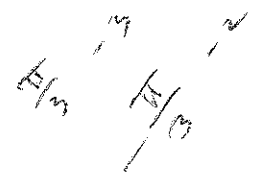
2. Define:  $y = \tan^{-1} x$  if and only if  $x = \tan y, -\frac{\pi}{2} < y < \frac{\pi}{2}$

3. Define:  $y = e^x$  if and only if  $x = \ln y$

4.  $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$



5.  $\cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$



6.  $\ln(e^5) = 5$

7.  $\ln(\ln e) = \ln 1 = 0$

8.  $\sec \frac{\pi}{3} = 2$

9.  $\frac{d}{dx} \sec \frac{x}{2} = \sec \frac{x}{2} \tan \frac{x}{2} \cdot \frac{1}{2}$

10.  $\frac{d}{dx} e^{2x-3} = e^{2x-3} \cdot 2$

ANS

1. cont.

$$11. \int \frac{1}{2x-1} dx = \frac{1}{2} \ln|2x-1| + C$$

$$12. \int \sec^2 x dx = \tan x + C$$

$$13. \int \frac{2}{1+x^2} dx = 2 \tan^{-1} x + C$$

$$14. \frac{d}{dx} \sec^2(x+1) = 2 \sec(x+1) \sec(x+1) \tan(x+1) = 2 \sec^2(x+1) \tan(x+1)$$

$$15. \frac{d}{dx} \ln 2 = 0$$

$$16. \frac{d}{dx} \sin^{-1} 3x = \frac{3}{\sqrt{1-9x^2}}$$

$$17. \int 3^{x+1} dx = \int e^{(x+1) \ln 3} dx = \frac{e^{(x+1) \ln 3}}{\ln 3} + C = \frac{3^{x+1}}{\ln 3} + C$$

$$18. \frac{d}{dx} \ln(x^3-1) = \frac{1}{x^3-1} 3x^2 = \frac{3x^2}{x^3-1}$$

$$19. \frac{d}{dx} \sqrt{1+\tan x} = \frac{1}{2} (1+\tan x)^{-1/2} \sec^2 x = \frac{\sec^2 x}{2\sqrt{1+\tan x}}$$

$$20. \int_0^2 \frac{2x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2) \Big|_0^2 = \frac{1}{2} \ln 5 - \frac{1}{2} \ln 1 = \frac{1}{2} \ln 5$$

II. 20 points

1. Find  $\frac{dy}{dx}$  if  $y = e^{3x} \cos x + \ln(\cos 2x)$ .

$$\begin{aligned} \frac{dy}{dx} &= e^{3x}(-\sin x) + \cos x e^{3x} \cdot 3 + \frac{1}{\cos 2x} (-\csc 2x \cdot \cot 2x \cdot 2) \\ &= -e^{3x} \sin x + 3 \cos x e^{3x} - \frac{2 \cot 2x}{\cos 2x} \end{aligned}$$

2. Find the arc length of the curve  $y = x^3 + 1$  between  $(-1, 0)$  and  $(0, 1)$ . Set up ONLY, do not carry out integration!

$$\begin{aligned} y' &= 3x^2 \\ AL &= \int_{-1}^0 \sqrt{1 + 9x^4} \, dx \end{aligned}$$

3. Use the trapezoidal rule to approximate  $\int_0^2 \frac{1}{1+x^3} \, dx$ . Use  $n = 5$  subintervals, and do not carry out final arithmetic!

$$\begin{aligned} x_0 &= 0 & y_0 &= 1 & \Delta x &= \frac{2}{5} = .4 \\ x_1 &= .4 & y_1 &= \frac{1}{1} \\ x_2 &= .8 \\ x_3 &= 1.2 \\ x_4 &= 1.6 \\ x_5 &= 2 \end{aligned}$$

$$\begin{aligned} T_5 &= .4 \left[ \frac{1}{2} + \frac{1}{1+(.4)^3} + \frac{1}{1+(.8)^3} + \frac{1}{1+(1.2)^3} + \frac{1}{1+(1.6)^3} \right. \\ &\quad \left. + \frac{1}{2} \frac{1}{9} \right] \\ \frac{1}{1+(\frac{2}{5})^3} &= \frac{1}{1+\frac{8}{125}} = \frac{125}{133} \end{aligned}$$

$$125 \overline{) 1125}$$