9:05
9:10
5 min
short.

Lots of people left with 15 min to go

med = 86

Math 231
Test II
Feb. 13, 1976

Name KEY

Show work!

I. 4 points ea.

1. Define: ln x = $\int_1^x \frac{1}{t}\,dt,\ x>0$

2. Define: $y = \tan^{-1}x$ if and only if $x = \tan y,\ -\frac{\pi}{2} < y < \frac{\pi}{2}$

3. Define: $y = e^x$ if and only if $x = \ln y$

4. $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

5. $\cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$

$\pi/3$ $\pi/3$

6. $\ln(e^5) = 5$

7. $\ln(\ln e) = \ln 1 = 0$

8. $\sec\frac{\pi}{3} = 2$

9. $\frac{d}{dx}\sec\frac{x}{2} = \sec\frac{x}{2}\tan\frac{x}{2}\cdot\frac{1}{2}$

10. $\frac{d}{dx}e^{2x-3} = e^{2x-3}\,2$

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I. cont.

11. $\int \frac{1}{2x-1}\,dx = \frac{1}{2}\ln|2x-1| + c$

12. $\int \sec^2 x\,dx = \tan x + c$

13. $\int \frac{2}{1+x^2}\,dx = 2\tan^{-1}x + c$

14. $\frac{d}{dx}\sec^2(x+1) = 2\sec(x+1)\sec(x+1)\tan(x+1) = 2\sec^2(x+1)\tan(x+1)$

15. $\frac{d}{dx}\ln 2 = 0$

16. $\frac{d}{dx}\sin^{-1}3x = \frac{3}{\sqrt{1-9x^2}}$

17. $\int 3^{x+1}\,dx = \int e^{(x+1)\ln 3}\,dx = \frac{e^{(x+1)\ln 3}}{\ln 3} + c = \frac{3^{x+1}}{\ln 3} + c$

18. $\frac{d}{dx}\ln(x^3-1) = \frac{1}{x^3-1}3x^2 = \frac{3x^2}{x^3-1}$

19. $\frac{d}{dx}\sqrt{1+\tan x} = \frac{1}{2}(1+\tan x)^{-1/2}\sec^2 x = \frac{\sec^2 x}{2\sqrt{1+\tan x}}$

20. $\frac{1}{2}\int_0^2 \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln(1+x^2)\Big|_0^2 = \frac{1}{2}\ln 5 - \frac{1}{2}\underbrace{\ln 1}_{0}$

$= \frac{1}{2}\ln 5$

II. 20 points

1. Find $\frac{dy}{dx}$ if $y = e^{3x}\cos x + \ln(\csc 2x)$.

$$\frac{dy}{dx} = e^{3x}(-\sin x) + \cos x\, e^{3x}\cdot 3 + \frac{1}{\csc 2x}(-\csc 2x \cot 2x \cdot 2)$$

$$= -e^{3x}\sin x + 3\cos x\, e^{3x} - 2\cot 2x$$

2. Find the arc length of the curve $y = x^3+1$ between $(-1,0)$ and $(0,1)$. Set up ONLY, do not carry out integration)!

$$y' = 3x^2$$

$$AL = \int_{-1}^{0} \sqrt{1+9x^4}\, dx$$

3. Use the trapezoidal rule to approximate $\int_0^2 \frac{1}{1+x^3}\, dx$. Use n = 5 subintervals, and do not carry out final arithmetic!

$x_0 = 0$ $\quad y_0 = 1$ $\quad \Delta x = \frac{2}{5} = .4$

$x_1 = .4$ $\quad y_1 = \frac{1}{}$

$x_2 = .8$

$x_3 = 1.2$

$x_4 = 1.6$

$x_5 = 2$

$$T_5 = .4\left[\frac{1}{2} + \frac{1}{1+(.4)^3} + \frac{1}{1+(.8)^3} + \frac{1}{1+(1.2)^3} + \frac{1}{1+(1.6)^3} + \frac{1}{2}\,\frac{1}{9}\right]$$

$\frac{1}{2}$

$$\frac{1}{1+(\frac{2}{5})^3} = \frac{1}{1+\frac{8}{125}} = \frac{125}{133}$$

$125\overline{)1125}$

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