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Math 231
Test III
March 5, 1976

Name KEY

8 min for just 9

a little easy

do all next time?

OMIT ONE PROBLEM. 11 points each.

1. a. If $\frac{dy}{dx} = 4y$, and $y = 1$ when $x = 1$, then $y = e^{-4}e^{4x} = e^{4x-4}$

$y = ce^{4x}$ ✓

$1 = ce^4$

$c = e^{-4}$

med = 8.4

b. If $\frac{dy}{dx} = y^2$, and $y = 1$ when $x = 1$, then $y = \frac{-1}{x-2}$

$y^{-2}dy = dx$

$-y^{-1} = x + c$

$-\frac{1}{y} = x + c$

$-1 = 1 + c$

$c = -2$

$-\frac{1}{y} = x - 2$

2. A bacteria culture has a count of 1000 which increases to a count of 3500 in one hour. If the rate of growth continues to be proportional to the count, what will be the count at the end of t hours (from the start)? When did the count double? (first time).

$y = Ce^{kt}$

$y = 1000e^{kt}$

$3500 = 1000e^{k}$

$k = \ln 3.5$

$y = 1000e^{(\ln 3.5)t}$

$2 = e^{\ln(3.5)t}$

$\ln 2 = \ln(3.5)t$

$t = \cancel{\frac{\ln 3.5}{\ln 2}} \quad \frac{\ln 2}{\ln 3.5}$

3. Write out (DO NOT solve) the partial fraction expansion of:

a. $\frac{x+1}{(x-1)(x+3)^3} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2} + \frac{D}{(x+3)^3}$

b. $\frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

c. $\frac{x}{x^2(x^2+4)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2}$

4. $\displaystyle\int x \ln x \, dx =$ (Integration by parts)

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$

$$= \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\frac{1}{x}\,dx$$

$$= \frac{x^2}{2}\ln x - \int \frac{x}{2}dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

5. $\displaystyle\int \frac{1}{(x-1)(x+5)}\,dx =$ (Partial fractions)

$$\frac{1}{(x-1)(x+5)} = \frac{A}{x-1} + \frac{B}{(x+5)} \qquad = \frac{1}{6}\int\frac{1}{x-1}dx - \frac{1}{6}\int\frac{1}{x+5}dx$$

$1 = A(x+5) + B(x-1) \qquad = \frac{1}{6}\ln|x-1| - \frac{1}{6}\ln|x+5| + C$

$x=1$

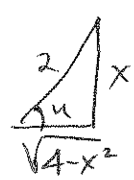
$1 = 6A+0 \quad A = \frac{1}{6}$

$x=-5$

$1 = 0 - 6B$

$B = -\frac{1}{6}$

6. $\displaystyle\int \frac{1}{(4-x^2)^{3/2}}\,dx = \int\frac{2\cos u}{2^3\cos^3 u}\,du$ (Trig. substitution)



$\frac{1}{4}\int \sec^2 u\,du$

$= \frac{1}{4}\tan u + c$

$\sin u = \frac{x}{2}$

$\cos u\,du = \frac{1}{2}dx \qquad = \frac{1}{4}\,\frac{x}{\sqrt{4-x^2}} + C$

$\frac{\sqrt{4-x^2}}{2} = \cos u$

$\sqrt{4-x^2} = 2\cos u$

7. $\displaystyle\int \frac{\cos x}{\sin x - 1}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sin x - 1| + C$

$u = \sin x - 1$

$du = \cos x\,dx$

8. $\displaystyle\int \sec^2 x \tan^2 x\,dx = \int u^2\,du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$

$u = \tan x$

$du = \sec^2 x\,dx$

③ $u = 1 + x^2$

$du = 2x\,dx$

$x^2 = u - 1$

$\displaystyle\int (u-1)\sqrt{u}\,\tfrac{1}{2}\,du$

$\displaystyle = \frac{1}{2}\int u^{3/2} - u^{1/2}\,du$

$\displaystyle = \frac{u^{5/2}}{5} - \frac{u^{3/2}}{3} + C$

$\displaystyle = \frac{(1+x^2)^{5/2}}{5} - \frac{(1+x^2)^{3/2}}{3} + C$

9. $\displaystyle\int x^3\sqrt{1+x^2}\,dx = \frac{x^2(1+x^2)^{3/2}}{3} - \int \frac{2}{3}\,x(1+x^2)^{3/2}\,dx$

①

$u = x^2$ $\quad dv = x\sqrt{1+x^2}\,dx$

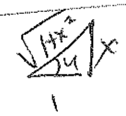
$du = 2x\,dx$ $\quad \displaystyle V = \frac{1}{2}\,\frac{(1+x^2)^{3/2}}{3/2}$

$\displaystyle = \frac{(1+x^2)^{3/2}}{3}$

$\displaystyle = \frac{x^2(1+x^2)^{3/2}}{3} - \frac{1}{3}\,\frac{(1+x^2)^{5/2}}{5/2} + C$

$\displaystyle = \frac{x^2(1+x^2)^{3/2}}{3} - \frac{2}{15}(1+x^2)^{5/2} + C$

②



$\tan u = x$

$\sec^2 u\,du = dx$

$\sqrt{1+x^2} = \sec u$

$\displaystyle\int = \int \tan^3 u \sec u \sec^2 u\,du$

$\displaystyle = \int \tan^3 u \sec^3 u\,du$

$\displaystyle = \int (-1+\sec^2 u)(\sec^2 u)\sec u \tan u\,du$

$V = \sec u$ $\quad \displaystyle\int (-1+v^2)v^2\,dv = \int -v^2 + v^4\,dv = -\frac{v^3}{3} + \frac{v^5}{5} + C$

$\displaystyle = -\frac{\sec^3 u}{3} + \frac{\sec^5 u}{5} + C$

$\displaystyle = -\frac{(1+x^2)^{3/2}}{3} + \frac{(1+x^2)^{5/2}}{5} + C$

10. $\int \frac{2}{x^2+2x+5}\,dx = \int \frac{1/2}{\left(\frac{x+1}{2}\right)^2+1}\,dx = \int \frac{1}{u^2+1}\,du = \tan^{-1}u + C$

$= \tan^{-1}\left(\frac{x+1}{2}\right) + C$

$$\frac{2}{x^2+2x+5} = \frac{2}{x^2+2x+1+4} = \frac{2}{(x+1)^2+4}$$

$$= \frac{\cancel{2}\,1/2}{\left(\frac{x+1}{2}\right)^2+1}$$

$u = \frac{x+1}{2}$

$du = \frac{1}{2}\,dx$