Math 232
Test I
October 3, 1969

1. Find the Cartesian coordinates of all points of intersection of the graphs of $r = 3 \sin \theta$ and $r = 2 \cos \theta$ .

2. Find polar coordinates for the points with cartesian coordinates
   a) $(-1, -\sqrt{5})$, b) $(\pi, -\pi)$ .

3. Sketch the graph of

$$r = \frac{5}{2 + 4\cos \theta}$$

   giving the coordinates of all vertices and foci.

4. What is the slope of the tangent line to the curve $r = 2\cos 4\theta$ at the point $(-1, \frac{\pi}{6})$.

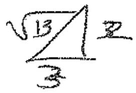
5. Find the height of the ellipse

$$r = \frac{2}{2 - \cos \theta} \quad .$$

6. Find the area within the cardioid $r = 1 - 2\sin\theta$ , but outside the inside loop (set up but do not integrate).

1. $r = 3\sin\theta \quad = 2\cos\theta$

$\tan\theta = \frac{2}{3}$



2,3 12 pts

$\sin\theta = \frac{2}{\sqrt{13}} \qquad \cos\theta = \frac{3}{\sqrt{13}}$

$x = 3\sin\theta\cos\theta = \frac{3\cdot 2\cdot 3}{13} = \frac{18}{13}$

$y = 3\sin\theta\sin\theta = \frac{3\cdot 4}{13} = \frac{12}{13}$

2. (a) $(-1, -\sqrt{3})$

$r^2 = 1+3 = 4$

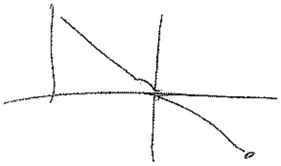
$r = 2$

$x \quad -1 = 2\cos\theta \qquad \cos\theta = -\frac{1}{2}$

$-\sqrt{3} = 2\sin\theta \qquad \sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = \frac{4\pi}{3} \qquad (2, \frac{4\pi}{3})$ 240°

(b) $(\pi, -\pi)$



$r^2 = 2\pi^2$

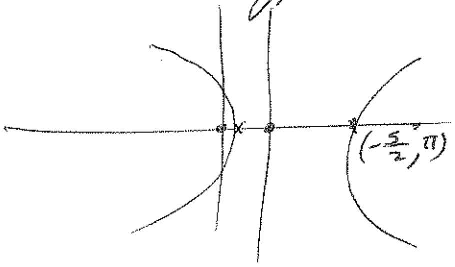
$r = \sqrt{2}\pi$

$(\sqrt{3}\pi, -\frac{\pi}{4})$ or $(\sqrt{2}\pi, \frac{7\pi}{4})$ 315°

3. $r = \frac{5}{2+4\cos\theta} = \frac{5/2}{1+2\cos\theta}$

$e = 2$ hyperbola $\quad V_1 \quad \theta = 0 \quad r = \frac{5}{2+4} = \frac{5}{6}$

$V_2 \quad \theta = \pi \quad r = \frac{5}{2-4} = \frac{5}{-2} = -\frac{5}{2}$



$F_1$ pole

$F_2 \quad -\frac{5}{2} + \frac{5}{6} = -\left(\frac{15+5}{6}\right) = -\frac{20}{6} = -\frac{10}{3}$

$-4 \quad (\frac{10}{\ }) \pi) \quad (10$

4. $r = 2\cos 4\theta \quad \left(-1, \frac{\pi}{6}\right)$

$y = 2\cos 4\theta \sin\theta$

$D_\theta y = 2\cos 4\theta\cos\theta - 8\sin 4\theta\sin\theta$

$\theta = \pi/6 \quad D_\theta y = 2\cos\frac{2\pi}{3}\cos\frac{\pi}{6} - 8\sin\frac{2\pi}{3}\sin\frac{\pi}{6}$

$= \cancel{2}\left(-\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \cancel{8}^{2}\frac{\sqrt{3}}{\cancel{2}}\left(\frac{1}{\cancel{2}}\right)$

$= -\frac{\sqrt{3}}{2} - 2\sqrt{3} = -\frac{5}{2}\sqrt{3}$

$\cancel{\theta}\ x = 2\cos 4\theta\cos\theta$

$D_\theta x = -2\cos 4\theta\sin\theta \cancel{+} 8\sin 4\theta\cos\theta$

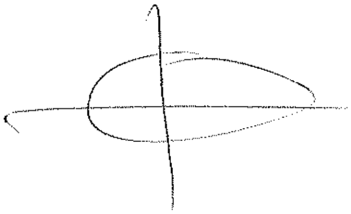
$\theta = \pi/6 \quad = -2\cos\frac{2\pi}{3}\sin\frac{\pi}{6} - 8\sin\frac{2\pi}{3}\cos\pi/6$

$= -\cancel{2}\left(-\frac{1}{2}\right)\left(\frac{1}{\cancel{2}}\right) - \cancel{8}^{2}\frac{\sqrt{3}}{\cancel{2}}\,\frac{\sqrt{3}}{\cancel{2}}$

$= +\frac{1}{2} - 6 = -\frac{11}{2}$

slope $= \frac{5\sqrt{3}}{11}$ ✓

5. $r = \frac{2}{2 - \cos\theta}$



slope = 0

$y = \left(\frac{2}{2-\cos\theta}\right)\sin\theta$

$D_\theta y = \frac{(2-\cos\theta)2\cos\theta - \cancel{(\text{[illegible]})}^{2}\sin\theta\sin\theta}{(2-\cos\theta)^2}$

$= 4\cos\theta - 2\cos^2\theta - 2\sin^2\theta = 0$

$4\cos\theta - 2 = 0$

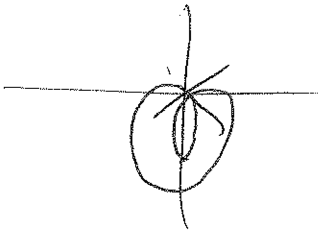
$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3} \qquad r = \frac{2}{2-\frac{1}{2}} = \frac{2}{3/2} =$

$y = \frac{4}{3}\sin\frac{\pi}{3} = \frac{\cancel{4}^{2}}{3}\frac{\sqrt{3}}{\cancel{2}} = \frac{2\sqrt{3}}{3}$

height $= \frac{4\sqrt{3}}{3}$

6.



$7 - 2\sin\theta = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \quad \frac{5\pi}{6}$

$$\int_{5\pi/6}^{\frac{13\pi}{6}} \left(\frac{1-2\sin\theta}{2}\right)^2 d\theta - \int_{\pi/6}^{5\pi/6} \left(\frac{1-2\sin\theta}{2}\right)^2 d\theta$$