Math 232
Test II
Dec. 2, 1969

Name Key

Part I: No partial credit.

1. $\sum_{k=1}^{\infty} (1 + \frac{1}{k})$ Converges( ) Diverges(✓)

2. $\sum_{k=1}^{\infty} \frac{6}{k^4}$ Converges(✓) Diverges( )

3. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ Converges(✓) Diverges( )

4. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ Converges( ) Diverges(✓)

3 2pts

5. $\sum_{k=1}^{\infty} \frac{2}{3^k} = 2\left(\frac{1}{1-\frac{1}{3}}\right) - 2$. $= 2\left(\frac{3}{2}\right) - 2 = 3 - 2 = 1$

6. $\lim_{n\to\infty} \frac{n^2+n+1}{2n^2-n-1} = \frac{1}{2}$.

Part II: Test for convergence (absolute, conditional, or divergence). Show work and identify tests used.

7. $\sum_{k=1}^{\infty} \frac{1}{2^n + n}$.

$\frac{1}{2^n+n} \le \frac{1}{2^n}$ converges (geometric series)

comparison test

8. $\sum_{k=1}^{\infty} \frac{(-1)^k}{2k-1}$

$\frac{1}{2k-1}$ decreases, limit 0

so converges by AST

$\frac{1}{2k-1} \ge \frac{1}{2k}$ diverges (comparison)

so converges conditionally

9. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

$\frac{(-1)^n (3)^n}{n!}$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{3^{k+1}}{(k+1)!} \cdot \frac{k!}{3^k} = \frac{3}{k+1} \to 0$ converges abs by Ratio test

10. $\sum_{k=1}^{\infty} \frac{k+1}{k^3-k+1}$

$\frac{k+1}{k^3-k+1} \le \frac{2k}{k^3} = \frac{2}{k^2}$ converges comparison test

$k \geq 8$

converges abs.

11. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$

$\int_2^{\infty} \frac{1}{(\ln x)} \frac{1}{x} dx = \int_{x=2}^{\infty} \frac{1}{u} du = \int_{\ln 2}^{\infty} \frac{1}{u} du$

$u = \ln x$

exists converges

converges abs

Part III:

12. For what values of x does the following series converge absolutely, converge conditionally, or diverge?

$\sum_{k=1}^{\infty} \frac{k}{3^k} (x-1)^k$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{k+1}{3^{k+1}} \cdot \frac{3^k}{k} |x-1| = \frac{k+1}{k} \frac{|x-1|}{3} \longrightarrow \frac{|x-1|}{3}$

$\frac{|x-1|}{3} < 1 \quad -3 < x-1 < 3$

$-2 < x < 4$ abs con

$x > 4, \ x < -2$ div

$x = 4 \quad \sum_{k=1}^{\infty} k$ div $\quad x = -2 \quad \sum_{k=1}^{\infty} (-1)^k k$ div

abs convergence $-2 < x < 4$

divergence otherwise

13. Write Taylor's Series for $f(x) = \frac{1}{(1+x)^3}$ about 2.

$f(x) = (1+x)^{-3} \qquad f(2) = 3^{-3}$

$f'(x) = -3(1+x)^{-4} \qquad f'(3) = \frac{-3}{3^4}$

$f''(x) = 12(1+x)^{-5} \qquad = \frac{12}{3^5}$

$\vdots$

$f^{(k)}(x) = (-1)^k (k+2) \cdots 3 (1+x)^{-k-3}$

$= \frac{(k+2)!}{2} (1+x)^{-k-3}$

$\sum_{k=0}^{\infty} \frac{(k+2)(k+1)}{2 \, 3^{k+3}} (x-2)^k$