95 pts $\mu = 78.2$ med = 81

Few had left by 40 min, but only half left at 55 min. Some checking answers

MATH 232
Test I
February 1, 1985

NAME Key

Calculators allowed

8 min

Time OK
Maybe short

(20) 1. Compute the following derivatives

a. $D_x(e^{2x} + \arcsin\frac{x}{3}) = e^{2x}\cdot 2 + \frac{1}{\sqrt{1+(\frac{x}{3})^2}}\cdot\frac{1}{3} = 2e^{2x} + \frac{1}{\sqrt{9-x^2}}$

b. $D_x(e^{-3x^3} + e^2) = e^{-3x^3}(-9x^2) + 0$

c. $D_x\frac{\sqrt{e^{2x}+5}}{(e^{2x}+5)^{1/2}} = \frac{1}{2}(e^{2x}+5)^{-1/2}\cdot e^{2x}\cdot 2 = \frac{e^{2x}}{\sqrt{e^{2x}+5}}$

d. $D_x(3^{2x} + \log_{10}x^2) = e^{\ln 3\cdot 2x}(2\ln 3) + \frac{1}{x\ln 10} = 3^{2x}2\ln 3 + \frac{1}{x\ln 10}$

$e^{\ln 3\,2x} + \frac{\ln x}{\ln 10}$

1:30

(30) 2. Evaluate the following integrals

a. $\int_0^1 e^{3x}dx = \frac{e^{3x}}{3}\Big|_0^1 = \frac{e^3}{3} - \frac{e^0}{3} = \frac{e^3}{3} - \frac{1}{3}$

1 late

25

b. $\int x e^{x^2-8}\,dx =$ $= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$

$u = x^2 - 8$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}e^{x^2-8} + C$

c. $\int_0^1 \frac{3}{\sqrt{1-x^2}}\,dx =$ $3 \arcsin x\Big|_0^1 = 3\arcsin 1 - 3\arcsin 0$

$= 3\cdot\frac{\pi}{2} - 0 = \frac{3}{2}\pi$

d. $\int \frac{1}{2x^2+8}\,dx =$

$\frac{1}{2}\int \frac{1}{x^2+4}\,dx = \frac{1}{2}\int \frac{1/4}{\left(\frac{x}{2}\right)^2+1}\,dx = \frac{1}{8}\arctan\frac{x}{2} + C$

e. $\int \frac{x+1}{x^2+1}\,dx =$ $\int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$

$= \frac{1}{2}\ln(x^2+1) + \arctan x + C$

4

(5) 3. Sketch the graph of $y = \arcsin(x+1)$ on the last page!!

25

430

(5) 4. Find $\lim_{x \to 0} \frac{\arctan x}{e^x - 1}$ $= \lim_{x \to 0} \frac{\frac{1}{1+x^2}}{e^x} = \frac{1}{1} = 1$

4 !as

(10) 5. On the last page the graph of a function is given

2 a. $f^{-1}(1) = 0$ $\qquad f^{-1}(2) = 3$

2 b. the domain of f^{-1} is: $[0, 2]$

6 c. On the last page, sketch the graph of $y = f^{-1}(x)$

hard a lot of to the work this.

5; 30

(10) 6. Let $f(x) = x^5 + 3x^3 - 2$ 32

$f(1) = 1 + 3 - 2 = 2$
$f^{-1}(2) = 1$

a. Find the slope of the line tangent to the graph of $y = f^{-1}(x)$ at the point (2, 1)

2 $f'(x) = 5x^4 + 9x^2$

6 $y = 1 \qquad f^{-1}(2) = \frac{1}{f'(1)} = \frac{1}{14}$

b. How do we know that this function has an inverse?

4 $f'(x) \geq 0$ so increasing

25

6.

(5) 7. Write an expression for A(t) = amount in an account after t years where the annual interest rate is 5% and compounding is done continuously, and the principal at the beginning was $500.

$$A(t) = 500e^{.05t}$$

6: 20

(15) 8. A bacteria culture growing under ideal conditions will grow at a rate proportional to the amount present. Suppose that at the beginning, the culture had a count of 10,000, and after 2 hours, the count was 30,000.

a. Derive a function f(t) which gives the count present after t hours.

b. What will be the count after 3 hours?

c. How many hours did it take for the count to reach 100,000?

$$f(x) = Ce^{kt}$$

$$= 10{,}000\, e^{kt}$$

$$30{,}000 = 10{,}000\, e^{k2}$$

$$3 = e^{k2}$$

$$\ln 3 = k2$$

$$k = \tfrac{1}{2}\ln 3$$

$$f(x) = 10{,}000\, e^{\frac{1}{2}\ln 3\, t}$$

$$f(3) = 10{,}000\, e^{\frac{1}{2}\ln 3\,(3)}$$

$$= 51{,}961.52$$

$$100{,}000 = 10{,}000\, e^{\frac{1}{2}\ln 3\, t}$$

$$10 = e^{\frac{1}{2}\ln 3\, t}$$

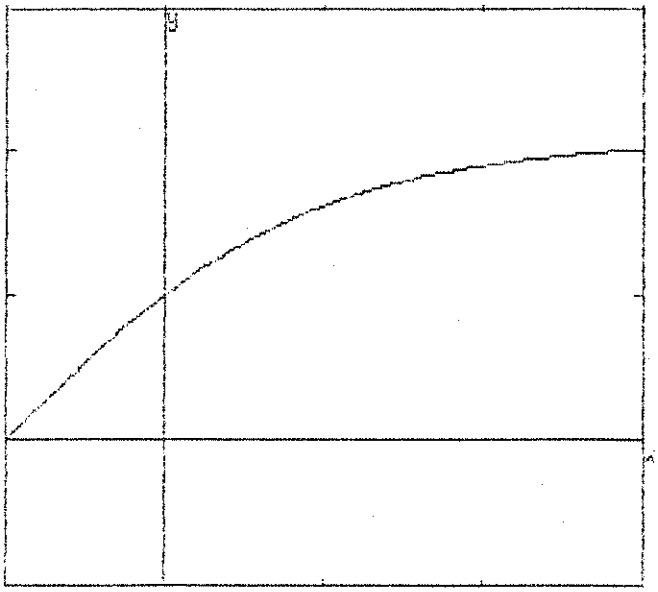
$$\ln 10 = \tfrac{1}{2}\ln 3\, t$$

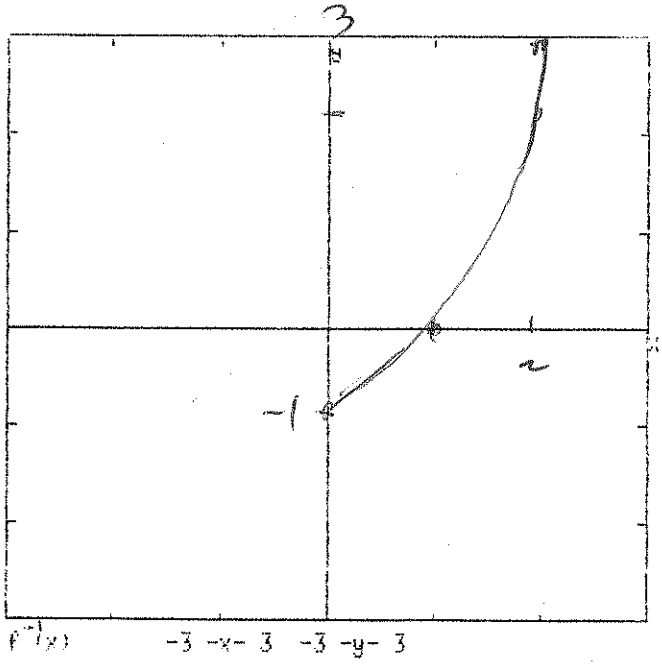
$$t = \frac{2\ln 10}{\ln 3}$$

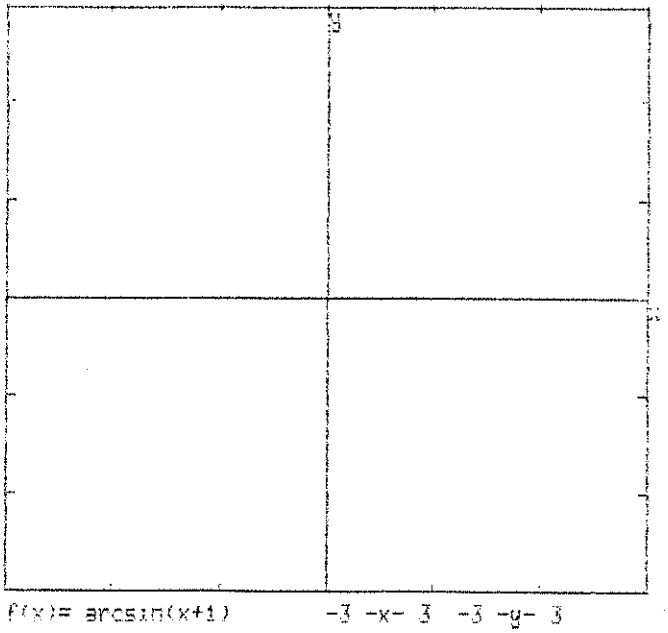
$$= 4.192$$

easy page.

20


y
f(x) -1 -x- 3 -1 -y- 3


3
y
2
-1
$f^{-1}(x)$ -3 -x- 3 -3 -y- 3


y
f(x)= arcsin(x+1) -3 -x- 3 -3 -y- 3