First left after 26 (lisa)
Complete after about 35 m
Time seems OK
2/3 still there after 50 min

MATH 232
Test II
February 22, 1985

Name ______________________

Show work. Calculators allowed, but show exact answers.

(7) 1. Find the solution to the differential equation

$$y' = e^y \sin x, \quad y = 1 \text{ when } x = 0.$$

$\frac{dy}{dx} = e^y \sin x$

$e^{-y}\,dy = \sin x\,dx$

$-e^{-y} = -\cos x + c$

$-e^{-1} = -1 + c$

$c = 1 - e^{-1}$

$\boxed{-e^{-y} = -\cos x + 1 - e^{-1}}$

or $\boxed{e^{-y} = \cos x + e^{-1} - 1}$

$\boxed{y = -\ln(\cos x + e^{-1} - 1)}$

(30) 2. Find

a. $D_x \sinh 2x = 2\cosh 2x$

b. $\int (\cosh x)^2 \sinh x\,dx = \int u^2\,du = \frac{u^3}{3} + c$

$u = \cosh x$

$du = \sinh x$

$= \frac{(\cosh x)^3}{3} + C$

Lot of Q about wording here.

(8) 3. Give the identities similar to $\cos^2 x + \sin^2 x = 1$ for

a. $\sec x$, $\tan x$ $\qquad 1 + \tan^2 x = \sec^2 x$

b. cosh sinh
$\cos h\ x\ \sin h\ x$ $\qquad \cosh^2 x - \sinh^2 x = 1$

(15) 4. Write out the form of the partial fractions for each of the following. (Do not solve for the constants).

a. $\dfrac{2}{(x-1)(x+2)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x+2)} + \dfrac{C}{(x+2)^2}$

b. $\dfrac{x^2+1}{x^2-1} = 1 + \dfrac{2}{x^2-1} = 1 + \dfrac{A}{x-1} + \dfrac{B}{x+1}$

$$\begin{array}{r} 1 \\ x^2-1\,\overline{)\,x^2+1} \\ \underline{x^2-1} \\ 2 \end{array}$$

c. $\dfrac{x-3}{(x-1)^2(x^2+3)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{Cx+D}{x^2+3} + \dfrac{Ex+F}{(x^2+3)^2}$

(60) 5. Integrate the following:

a. $\displaystyle\int \frac{x}{x^3 - x}\,dx = \int \frac{.5}{x-1} - \frac{.5}{x+1}\,dx = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$

$$\frac{\cancel{x}}{\cancel{x}(x^2-1)} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$1 = A(x+1) + B(x-1)$$

$x = 1 \qquad 1 = 2A \qquad A = .5$

$x = -1 \qquad 1 = -2B \qquad B = -.5$

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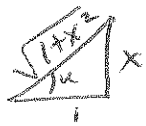
b. $\int \frac{1}{(1+x^2)^{3/2}}\,dx =$ $\int \frac{1}{\sec^3 u}\sec^2 u\,du = \int \frac{1}{\sec u}\,du$

(Hint: Try a Trig. substitution)



$= \int \cos u\,du = \sin u + C$

$= \boxed{\frac{x}{\sqrt{1+x^2}} + C}$

$\tan u = x$

$\sec^2 u\,du = dx$

$\sqrt{1+x^2} = \sec u$

c. $\int xe^{3x}\,dx =$ $= \frac{x}{3}e^{3x} - \int \frac{1}{3}e^{3x}\,dx = \boxed{\frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C}$

(Try parts)

$u = x \qquad dv = e^{3x}dx$

$du = dx \qquad v = \frac{1}{3}e^{3x}$

d. $\int (\sin x)^3 (\cos x)^2\,dx =$

$u = \cos x$

$du = -\sin x\,dx$

$\int \sin x\,(1-\cos^2 x)\cos^2 x\,dx$

$-\int \sin x\,(1-u^2)\,u^2\,du$

$\int u^4 - u^2\,du = \frac{u^5}{5} - \frac{u^3}{3} + C$

$= \boxed{\frac{(\cos x)^5}{5} - \frac{(\cos x)^3}{3} + C}$

$\frac{}{30}$

e. $\int 3x^2 e^x dx = \quad = 3x^2e^x - \int 6xe^x dx$

$u = 3x^2 \qquad dv = e^x dx$

$du = 6x\,dx \qquad v = e^x$

$= 3x^2e^x - \left[6xe^x - \int 6e^x dx\right]$

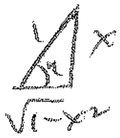
$$\boxed{= 3x^2e^x - 6xe^x + 6e^x + C}$$

$u = 6x \qquad dv = e^x dx$

$du = 6\,dx \qquad v = e^x$

f. $\int x^3 \sqrt{1 - x^2}\, dx = \quad = \int \sin^3 u \cos u \cos u\, du$

(1)



$\sin u = x$

$\cos u\, du = dx$

$\sqrt{1-x^2} = \cos u$

$= \int \sin u\,(1-\cos^2 u)\cos^2 u\, du$

$= \cancel{\int \sin u} - \int (1-v^2)v^2\, dv$

$v = \cos u$

$dv = \cancel{-\sin u}\, du$

$= \cancel{-} \int v^4 - v^2\, dv$

$= \frac{v^5}{5} - \frac{v^3}{3} + C$

$= \frac{(\cos u)^5}{5} - \frac{(\cos u)^3}{3} + C$

$= \frac{\left(\sqrt{1-x^2}\right)^5}{5} - \frac{\left(\sqrt{1-x^2}\right)^3}{3} + C$

(2) parts

$u = x^2 \qquad dv = x\sqrt{1-x^2}\,dx$

$du = 2x\,dx \qquad v = \frac{(1-x^2)^{3/2}}{3/2}\left(-\frac{1}{2}\right)$

$= -\frac{1}{3}(1-x^2)^{3/2}$

$-\frac{x^2}{3}(1-x^2)^{3/2} - \int -\frac{2}{3}x(1-x^2)^{3/2} dx$

$\ldots \quad -\frac{1}{3}\frac{(1-x^2)^{5/2}}{5/2} + C$

$= -\frac{x^2}{3}(1-x^2)^{3/2} - \frac{2}{15}(1-x^2)^{5/2} + C$

(3) sub $\quad u = 1-x^2 \qquad du = -2x\,dx$

$\int (1-u)\sqrt{u}\left(-\frac{1}{2}\right)du = -\frac{1}{2}\int \sqrt{u} - u^{3/2}\, du$

$= -\frac{1}{2}\left[\frac{u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2}\right] + C$

$-\frac{1}{3}(1-x^2)^{3/2} + \frac{1}{5}(1-x^2)^{5/2} + C$

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