1/2 left before
50 min

~ 10 min

Time OK

MATH 232
TEST III
MARCH 22, 1985

NAME ______________________

State clearly which convergence tests used.

(15) 1. Identify as convergent (absolute or conditional) or divergent.

a. $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges p series p > 1

b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ $\Sigma \frac{1}{n}$ div

alt ser ⇒ conv

cond conv

c. $\sum_{n=1}^{\infty} \sqrt{n}$ $\sqrt{n} \not\to 0$, div

? 20

(12) 2. Find the sum of the following infinite series if convergent, or identify as divergent.

a. $\sum_{n=0}^{\infty} \frac{2}{3^n}$ $= 2 \sum_{n=0}^{\infty} \cancel{2}\left(\frac{1}{3}\right)^n = 2 \frac{1}{1-\frac{1}{3}} = \cancel{\frac{6}{8}}\ 3$

40

21

b. $\sum_{n=2}^{\infty} (-1)^n (\frac{2}{3})^n$ $= \sum_{n=2}^{\infty} \left(-\frac{2}{3}\right)^n = \left(-\frac{2}{3}\right)^2 \sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n$

$= \frac{4}{9} \cancel{\frac{1}{1-2/3} = \frac{4}{3}}$ $\quad \frac{4}{9} \frac{1}{1+2/3} = \frac{4}{9}\frac{3}{5} = \frac{4}{15}$

or $\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n - 1 + \frac{2}{3} = \frac{3}{5} - 1 + \frac{2}{3} = \frac{9-15+10}{15} = \frac{4}{15}$

(8) 3. Use the integral test to determine if $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges or diverges.

$\int_1^{\infty} \frac{1}{\sqrt{x}} dx = \lim_{L\to\infty} \int_1^{L} x^{-1/2} dx = \lim_{L\to\infty} \left.\frac{x^{1/2}}{1/2}\right|_1^L = \lim_{L\to\infty} 2\sqrt{L} = \infty$

div.

2:

(14) 4. Test each of the following infinite series for convergence or divergence.

7 a. $\sum_{n=3}^{\infty} \frac{1}{n^3 - 2n^2}$ (use (limit) comparison test).

$$\frac{\frac{1}{n^3-2n^2}}{\frac{1}{n^3}} = \frac{n^3}{n^3-2n^2} = \frac{1}{1-2/n} \to 1$$

$\sum \frac{1}{n^3}$ conv $\Rightarrow$ $\sum \frac{1}{n^3-2n^2}$ conv

21

2:30

b. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$ (use alternating series test).

$\lim_{n\to\infty} \frac{n}{e^n} = \lim_{n\to\infty} \frac{1}{e^n} = 0$

7 $D\frac{e^n}{e^n} = \frac{e^n \cdot 1 - ne^n}{e^{2n}} = \frac{e^n(1-n)}{e^{2n}} < 0 \quad n>1$ dec

so conv AST.

3:30

(5) 5. Using infinite series, write the repeating decimal .262626... as a ratio of integers.

$$\sum_{n=1}^{\infty}(.26)(.01)^n = 26 \sum_{n\ge}(.01)^n = .26\sum_{n=0}^{\infty}(.01)^n$$

$$= .26 \frac{1}{1-.01} = \frac{.26}{.99} = \frac{26}{99}$$

5 $\sum_{n=1}^{\infty} 26(.01)^n$

$26\sum_{n=0}^{\infty}(.01)^n - 26$

$26 \frac{1}{.99} - 26$

4

(~~10~~ 11) 6. a. Write the first 3 terms of the Taylor series for $f(x) = \sqrt{x+1}$ about $a = 0$ (Mclaurin series).

b. Use this to approximate $\sqrt{1.1}$

c. Estimate the accuracy of this approximation.

$f(x) = (x+1)^{1/2}$ $\qquad f(0) = 1$

$f'(x) = \frac{1}{2}(x+1)^{-1/2}$ $\qquad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(x+1)^{-3/2}$ $\qquad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}(x+1)^{-5/2}$ $\qquad f'''(0) = \frac{3}{8}$

6 a. $1 + \frac{1}{2}\cdot\frac{1}{2}x + \frac{1}{4}\cdot\frac{1}{2}x^2 = 1 + \frac{1}{2}x - \frac{1}{8}x^2$

3 b. $f(.1) \approx 1 + (.5)(.1) - \frac{1}{8}(.1)^2 = 1 + .05 - .00125$

$.125 \qquad = 1.04875$

2 c. $err = \frac{f'''(\xi)x^3}{6} = \frac{\frac{3}{8}(\xi+1)^{-5/2}}{6}x^3 \le \frac{3}{48}x^3 = \frac{3}{48}(.1)^3$

$= .000625$

6:30

23

(25) 7. Test for convergence (absolute or conditional)

a. $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

$$\frac{n}{n^3+1} \le \frac{n}{n^3} = \frac{1}{n^2}$$

$\sum \frac{1}{n^2}$ conv $\Rightarrow$ CT $\sum \frac{n}{n^3+1}$ conv (abs)

b. $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

$\frac{n^2}{n^2+1} \to 1 \neq 0$ div

c. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n3^n}$

$\frac{1}{n3^n} \le \frac{1}{3^n}$ $\sum \left(\frac{1}{3}\right)^n$ conv

so by CT $\sum \frac{1}{n3^n}$ conv

$\Rightarrow$ $\sum (-1)^n \frac{1}{n3^n}$ conv abs.

d. $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\sqrt{n-1}}$

$\frac{1}{\sqrt{n-1}} \ge \frac{1}{\sqrt{n}}$ $\sum \frac{1}{\sqrt{n}}$ div $\Rightarrow$ CT $\sum \frac{1}{\sqrt{n-1}}$ div

$\frac{1}{\sqrt{n-1}} \to 0$, dec ,

by AST $\sum (-1)^n \frac{1}{\sqrt{n-1}}$ conv cond

25

(10) 8. For what values of x is the following power series absolutely convergent, conditionally convergent, and divergent:

$$\sum_{n=1}^{\infty} \frac{x^n}{2n}$$

$$\left| \frac{\dfrac{x^{n+1}}{2(n+1)}}{\dfrac{x^n}{2n}} \right| = \frac{n\,|x|}{n+1} \longrightarrow |x|$$

$|x| < 1$ cov abs

$|x| > 1$ ~~cov~~ div

$x = 1$ $\quad \sum \frac{1}{2n}$ div

$x = -1$ $\quad \sum \frac{(-1)^n}{2n}$ cov by AST

cov cond

$(-1, 1)$ cov abs

$[1, \infty)$ div

$(-\infty, -1)$ div

$x = -1$ cov cond

10

9+