Time OK

mean 73.1
med 73

MATH 252
Test IV
April 12, 1995

NAME KEY

NOTICE: Set up integrals, but do not carry out final integration. Calculators allowed.

(5) 1. For a point which has polar coordinates $(3, \frac{\pi}{4})$:

a. Give the cartesian coordinates.

$x = 3\cos\frac{\pi}{4} = \frac{3}{\sqrt{2}}$

$y = 3\sin\frac{\pi}{4} = \frac{3}{\sqrt{2}}$

$\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

b. Give another set of polar coordinates.

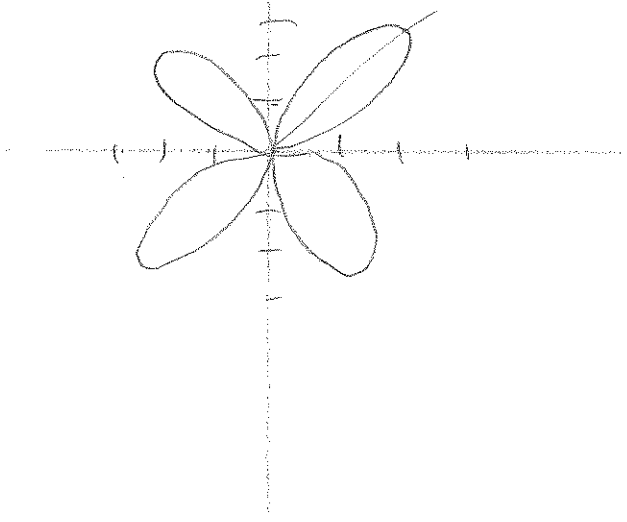
$\left(-3, \frac{5\pi}{4}\right)$

(15) 2. $r = 3\sin 2\theta$

a. Sketch the graph of this curve.

b. Find the area enclosed by this curve.

a



b $4\int_0^{\pi/2} \frac{1}{2}(3\sin 2\theta)^2 \, d\theta$

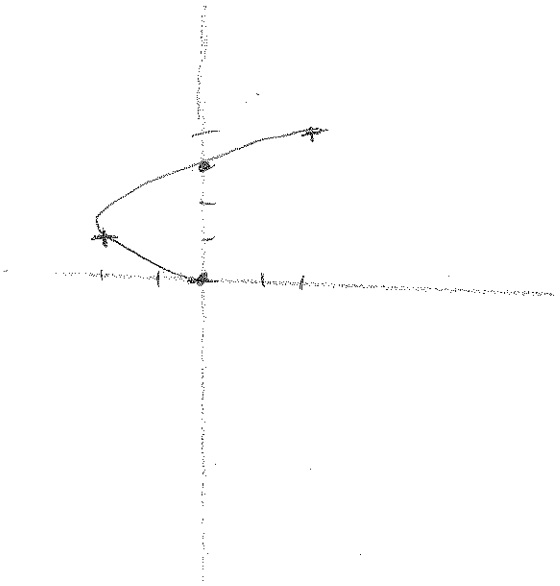
(20) 3. Let $x = t^3 - 3t$, $y = t^2$ for $0 \le t \le 2$.

A. Give coordinates of the end points and the points where it crosses the axes. (If it does).

B. What are the slopes of the tangent lines at the end points?

C. What is the length of this curve?

D. Sketch the graph of this curve.



A $t=0$ $x=0$ $y=0$

$t=2$ $x = 8-6 = 2$ $y=4$

$x=0$ $t(t^2-3)=0$

$t=0, \pm\sqrt{3}$

$t=\sqrt{3}$ $y=3$

$(0,3)$

$y=0$ $t=0$

$(0,0)$

B. $\frac{dy}{dt} = 2t$

$\frac{dx}{dt} = 3t^2-3$

$t=0$ $\frac{dy}{dx} = 0$

$t=2$ $\frac{dy}{dx} = \frac{4}{9}$

C $\int_0^2 \sqrt{(3t^2-3)^2 + (2t)^2}\, dt$

D $t=1$ $x=-2$

$y=$

(15) 4. Consider the cardioid $r = 1 + \cos\theta$ and the circle $r = 3\cos\theta$. Find the area

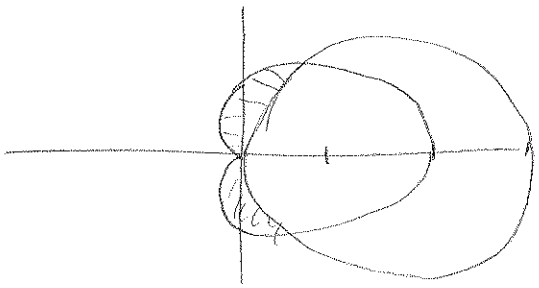
A. inside the cardioid but outside the circle.

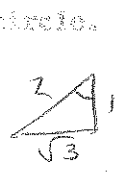
B. inside both curves.

$1 + \cos\theta = 3\cos\theta$

$1 = 2\cos\theta$

$\cos\theta = \frac{1}{2}, \quad \theta = \frac{\pi}{3}, -\frac{\pi}{3}$





A. $2\left[\int_{\pi/3}^{\pi} \frac{1}{2}(1+\cos\theta)^2 d\theta - \int_{\pi/3}^{\pi/2} \frac{1}{2}(3\cos\theta)^2 d\theta\right]$

B. $2\left[\int_{\pi/3}^{\pi/2} \frac{1}{2}(3\cos\theta)^2 d\theta + \int_{0}^{\pi/3} \frac{1}{2}(1+\cos\theta)^2 d\theta\right]$

(5) 5. Give a set of parametric equations for the curve whose cartesian equation is $x^2 + \frac{y^2}{9} = 1$.

$x = \cos t$

$\frac{y}{3} = \sin t$

$y = 3\sin t \qquad 0 \le t \le 2\pi$

(20) 6. Sketch the graphs of the following curves on attached graph paper. Give coordinates of foci, vertices, and draw asymptotes (if any).

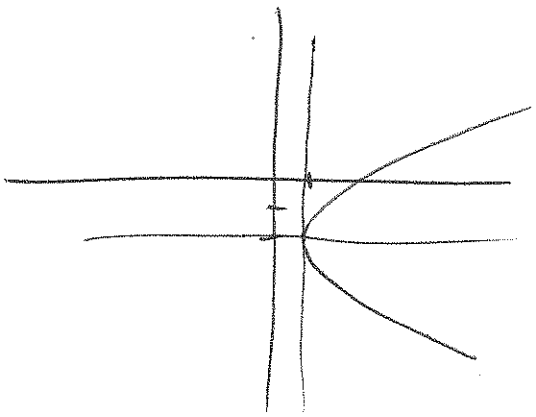
A. $2y^2 + 8y - x + 9 = 0$

B. $16x^2 - 9y^2 + 32x + 54y = 209$

$2(y^2 + 4y + \quad) = x - 9$

$2(y^2 + 4y + 4) = x - 9 + 8 = x - 1$

$2(y+2)^2 = x - 1$



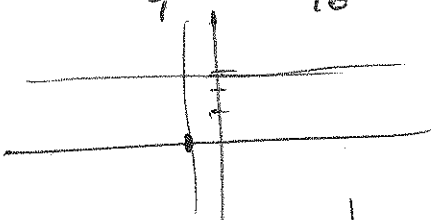
$16x^2 + 32x - 9y^2 + 54y = 209$

$16(x^2+2x+1) - 9(y^2 - 6y + 9)$

$= 209 + 16 - 81$

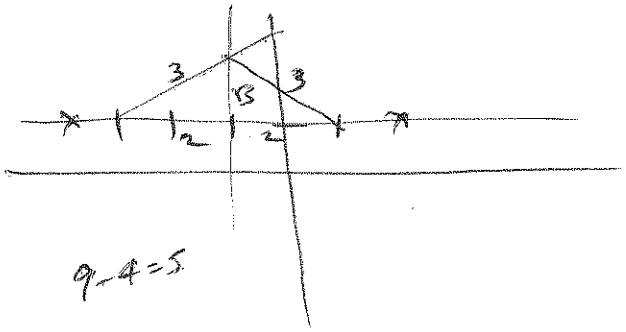
$16(x+1)^2 - 9(y-3)^2 = 144$

$\frac{(x+1)^2}{9} - \frac{(y-3)^2}{16} = 1$



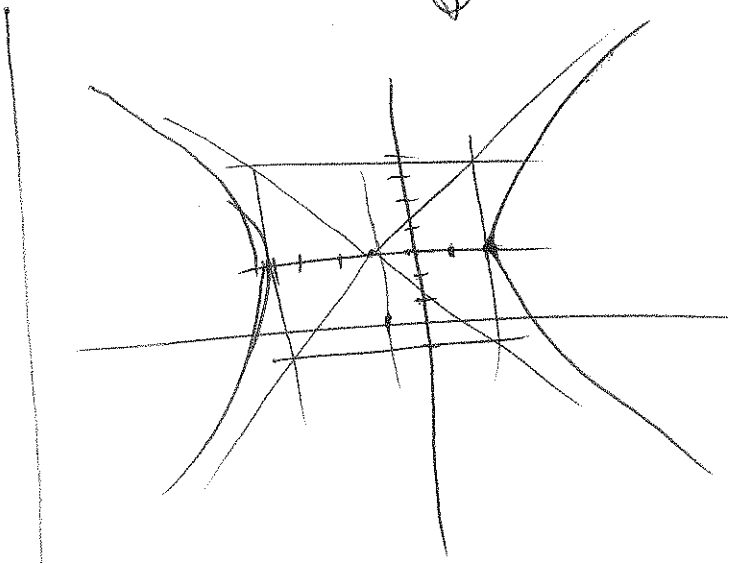
209
65
144
16
9
144

C. A hyperbola has foci at (1,1) and (-3,1), and vertices at (2,1) and (-4,1). Sketch this curve on graph paper, and find its equation.

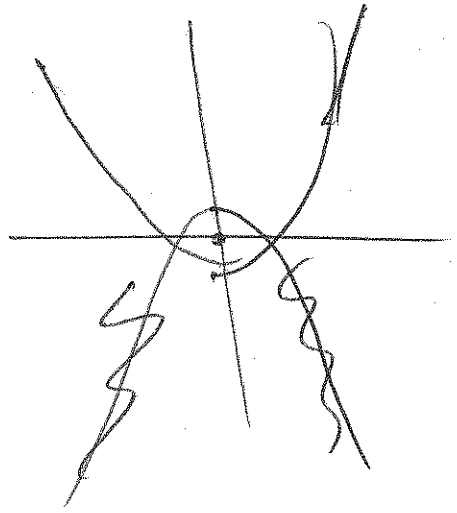


$9 - 4 = 5$

$\frac{(x+1)^2}{9} + \frac{(y-1)^2}{5} = 1$



(10) 8. Sketch the curve $r = \frac{2}{1 - \sin\theta}$ on graph paper and convert it to an equation in cartesian coordinates.



$ep = 2$

$1 - e$ and $e = 1$

parabola.

$r - r\sin\theta = 2$

$r - y = 2$

$\sqrt{x^2+y^2} = y + 2$

$x^2 + y^2 = y^2 + 4y + 4$

$x^2 = 4(y+1)$