Math 232
Test I
April 21, 1972 Name *Key*

(10) 1. Hand in the take-home problem.

(5) 2. State precisely the definition of an ellipse.

(25) 3. DO <u>TWO</u> of the following: Circle the ones you want counted.

a. Prove: If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then $\lim_{n\to\infty}(a_n+b_n) = a+b$.

b. Prove: A decreasing sequence which is bounded below has a limit.

c. Find the centroid (center of mass) of a semi-circle.

d. Find $\lim_{x\to\infty} \frac{p(x)}{e^x}$, where $p(x)$ is a polynomial of degree n (i.e. $p(x) = a_n x^n + \dots + a_1 x + a_0$; $a_0, a_1, \dots, a_n$ real numbers).

c.
$$f(x) = \sqrt{r^2-x^2}$$

$$M_x = \frac{\rho}{2}\int_{-r}^{r} f(x)^2\,dx = \frac{\rho}{2}\int_{-r}^{r}(r^2-x^2)\,dx = \frac{\rho}{2}\left(r^2x-\frac{x^3}{3}\right)\Big|_{-r}^{r}$$

$$= \frac{\rho}{\cancel{2}}\,\frac{2}{3}r^3 \cdot \cancel{2} = \frac{2r^3}{3}\rho$$

$$\bar{y} =$$

$$m = \rho\int_{-r}^{r}\sqrt{r^2-x^2}\,dx = \frac{\pi r^2}{2}\rho$$

$$\bar{y} = \frac{\frac{2r^3}{3}\rho}{\pi\frac{r^2}{2}\rho} = \frac{4r}{\pi 3}$$

$$\bar{x} = 0$$

(20) 4. Evaluate the following limits, if they exist.

a. $\lim_{n\to\infty} \frac{n^2+n+1}{4n^2-3} = \lim_{n\to\infty} \frac{1+\frac{1}{n}+\frac{1}{n^2}}{4-\frac{3}{n^2}} = \frac{1}{4}$

b. $\lim_{x\to 0} \frac{\ln x}{x^2} = -\infty$

c. $\lim_{x\to\frac{\pi}{2}} \frac{\cos^2 x}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}} \frac{-2\cos x \sin x}{1} = 0$

d. $\lim_{x\to\infty} (1-\frac{6}{x})^x = e^{-6}$

$e^{x\ln(1-\frac{6}{x})}$

$\lim_{x\to\infty} \frac{\ln(1-\frac{6}{x})}{1/x} = \lim_{x\to\infty} \frac{\frac{1}{1-\frac{6}{x}}\cdot\frac{-6}{x^2}}{1/x^2} = -6$

(20) 5. Evaluate the following integrals, if they converge.

a. $\int_3^\infty x e^{-2x}\,dx = \lim_{L\to\infty}\int_3^L x e^{-2x}\,dx = \lim_{L\to\infty}\left[-\frac{1}{2}xe^{-2x}\Big|_3^L - \int_3^L -\frac{1}{2}e^{-2x}\,dx\right]$

$u = x \quad dv = e^{-2x}dx$

$du = dx \quad v = -\frac{1}{2}e^{-2x}$

$= \lim_{L\to\infty}\left[-\frac{1}{2}Le^{-2L} + \frac{3}{2e^6} - \frac{1}{4}e^{-2L} + \frac{1}{4}e^{-6}\right]$

$= \frac{3}{2e^6} + \frac{1}{4e^6} = \frac{7}{4}e^{-6}$

b. $\int_{-1}^2 \frac{1}{x^2}\,dx = \int_{-1}^0 + \int_0^2$ DNE

$\int_{-1}^0 \frac{1}{x^2} = \lim_{t\to 0^-}\int_{-1}^t \frac{1}{x^2}\,dx = \lim_{t\to 0^-} -x^{-1}\Big|_{-1}^t = \lim(-\frac{1}{t}+1) = \text{DNE}$

$\int_0^2 \frac{1}{x^2}\,dx = \lim_{t\to 0^+}\int_t^2 \frac{1}{x^2}\,dx = \lim_{t\to 0^+} -x^{-1}\Big|_t^2 = \lim_{t\to 0^+}\left[-\frac{1}{2}+\frac{1}{t}\right] = -$

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5. $\int_0^{\pi/2} \sec^2 x \, dx = \lim_{t \to \frac{\pi}{2}^-} \int_0^t \sec^2 x \, dx = \lim_{t \to \frac{\pi}{2}^-} \tan x \Big|_0^t$

$\frac{1}{\cos^2}$

$= \lim_{t \to \frac{\pi}{2}^-} (\tan t - 0)$

$=$ DNE.

(10) 6. Find the center of mass of the following discrete system:

point	(-1,2)	(1,11)	(0,4)	(2,-3)
mass	3	4	2	4

$m = 13$

$M_x = 3(2) + 4(11) + 2(4) + 4(-3) = 6 + 44 + 8 - 12 = 46$

$6 \quad 44 \quad 50 \quad -4 \quad 46$

$\boxed{\bar{y} = \frac{46}{13}}$

$M_y = 3(-1) + 4(1) + 2(0) + 4(2) = -3 + 4 + 8 = 9$

$\boxed{\bar{x} = \frac{9}{13}}$

(10) 7. Carefully sketch the graph of the following equations, including foci, vertices (with coordinates), and asymptotes (with equations) when they apply.

$9x^2 - y^2 - 36x - 6y + 18 = 0$

$\cancel{x^2 \; 2y^2 \; 4x}$

$9(x^2 - 4x + 4) - (y^2 + 6y + 9) = -18 + 36 - 9$

$9(x-2)^2 - (y+3)^2 = 9$

$(x-2)^2 - \frac{(y+3)^2}{9} = 1$

$a = 1$

$b = 3$

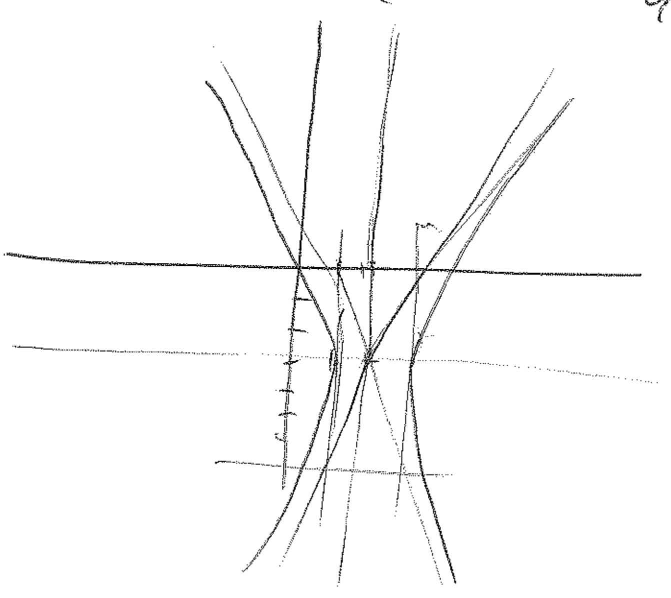
$c^2 = 1 + 9 = 10$

$c = \sqrt{10}$

Center $(2,-3)$

foci $(2+\sqrt{10}, -3)$ $(2-\sqrt{10}, -3)$

Vertex $(3,-3)$ $(1,-3)$



asymptotes

$\frac{y+3}{x-2} = 3$

$y = 3x - 6 - 3$

$= 3x - 9$

$\frac{y+3}{x-2} = -3$

$y = -3x + 6 - 3$

$= -3x + 3$