Math 232
Final Exam
June 11, 1975

Name KEY

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I. 5 points each.

1. The graph of $x^2+xy+y^2+x-5 = 0$ is a

$B^2-4AC = 1-4 = -3 < 0$ ellipse

2. What is the slope of the normal line to the curve $x^2-xy+y = 1$ at $(1,1)$?

not a curve

$2x - x\frac{dy}{dx} - y + \frac{dy}{dx} = 0$ $\quad -2\frac{dy}{dx} = -1$

OMIT OMPIRE

$2 - \frac{dy}{dx} - 1 + \frac{dy}{dx} = 0$ $\quad \frac{dy}{dx} = \frac{1}{2}$ +2

3. What is the angle of rotation that will eliminate the xy term in the expression $x^2+2xy-y^2$?

$\cot 2\alpha = \frac{A-C}{B} = \frac{1+1}{2} = 1$ $\quad 2\alpha = \frac{\pi}{4}$ $\quad \alpha = \frac{\pi}{8}$

4. $\frac{d}{dx}(\cosh(x^2+1)) = \sinh(x^2+1)\,2x$

5. $\int \tanh x\,dx = \int \frac{\sinh x}{\cosh x}\,dx = \ln(\cosh x) + C$

6. OMIT $\int \frac{\frac{1}{2}}{1+4x^2}\,dx = \frac{1}{2}\int \frac{1}{1+(2x)^2}\,2\,dx = \frac{1}{2}\tan^{-1}(2x) + C$

7. What is the slope of the line(s) tangent to the curve $r = 1+\cos\theta$ at the pole?

$1+\cos\theta = 0$ $\quad \cos\theta = -1$ $\quad \theta = \pi$ (0)

8. Find the cartesian coordinates for the point with cylindrical coordinates $(4, \frac{2\pi}{3}, 2)$.

$x = 4\cos\frac{2\pi}{3} = 4(-\frac{1}{2}) = -2$

$y = 4\sin\frac{2\pi}{3} = 4(\frac{\sqrt{3}}{2}) = 2\sqrt{3}$ $\quad 3\sqrt{}$

$(-2, 2\sqrt{3}, 2)$

9. $\vec{i}+\vec{j}+\vec{k} + 2(\vec{i}-2\vec{j}) =$

$3\vec{i} - 3\vec{j} + \vec{k}$

10. $(-\vec{i}+2\vec{j}-\vec{k})\cdot(2\vec{i}-\vec{j}-\vec{k}) = -2-2+1 = -3$

5b

I. cont.

11. $(i+j-k)\times(2i-j-k) = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & -1 & -1 \end{vmatrix} \begin{matrix} i & j \\ 1 & 1 \\ 2 & -1 \end{matrix} = -i - 2j - k - 2k - i + j$

$= -2i - j - 3k$

12. The unit tangent vector to the curve $\vec{R} = (t+1)\vec{i} + (t^2-1)\vec{j}$ at (3,3) is

$\frac{d\vec{R}}{dt} = i + 2tj \qquad \vec{T} = \frac{1}{\sqrt{17}}i + \frac{4}{\sqrt{17}}j$

$t=2$

$3\,3 \quad = i + 4j$

$|\ | = \sqrt{1+16} = \sqrt{17}$

II. 10 points ea.

1. An air plane is flying along the path $x = 100t$, $y = 2t^2$, $z = t^2$, where t is in hours.

a. Find the velocity and acceleration;

b. " " " " " of the "shadow" (in xy plane);

$\vec{R} = 100t\,i + 2t^2 j + t^2 k$

$\vec{v} = 100i + 4tj + 2tk$

$\vec{a} = 4j + 2k$

$100i + 4tj$

$4j$

2. A particle is moving at a constant speed along the straight line $y = 2x + 3$. At one instant it was at (1,5) (t=0) and 3 seconds later at (3,9). Find parametric equations for the position of the particle in terms of the time parameter.

$\vec{PH} = 2i + 4j$

$\vec{M} = \frac{2}{3}i + \frac{4}{3}j$

$\vec{R} = i + 5j + t\left(\frac{2}{3}i + \frac{4}{3}j\right)$

$= \left(\frac{2}{3}t+1\right)i + \left(\frac{4}{3}t+5\right)j$

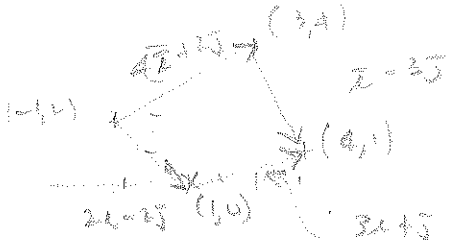
$x = \frac{2}{3}t + 1$

$y = \frac{4}{3}t + 5$

II. cont.

3. Is the quadrilateral with vertices (-1,2), (3,4), (1,0), and (4,1) a square? a rectangle? a trapezoid?

none of these



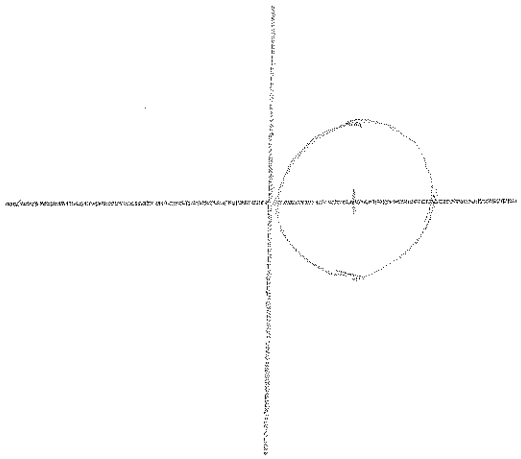
III. Graph each of the following in the plane: 10 points each.

1. $x^2+y^2-2x = 0$

$x^2-2x+1+y^2 = 1$

$(x-1)^2+y^2 = 1$

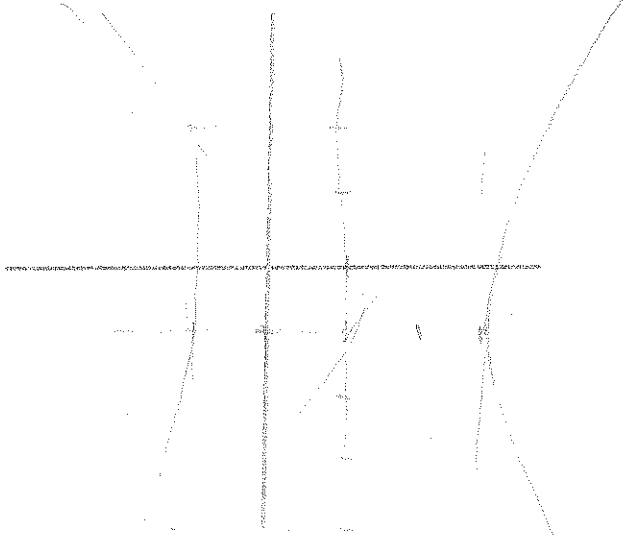
circle
ctr (1,0)
rad 1



2. $9(x-1)^2-4(y+1)^2 = 36$

$\frac{(x-1)^2}{4} - \frac{(y+1)^2}{9} = 1$

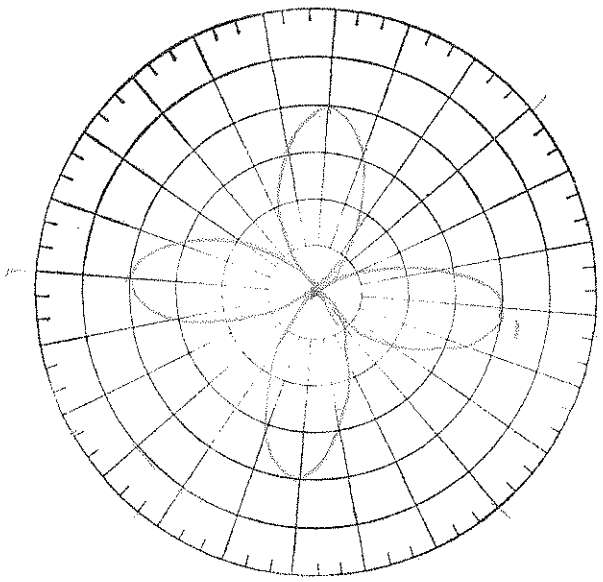
hyperbola
ctr (1,-1)
$a = 2$
$b = 3$



III. cont.

3. $r = \cos 2\theta$

θ	r
0	1
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	-1



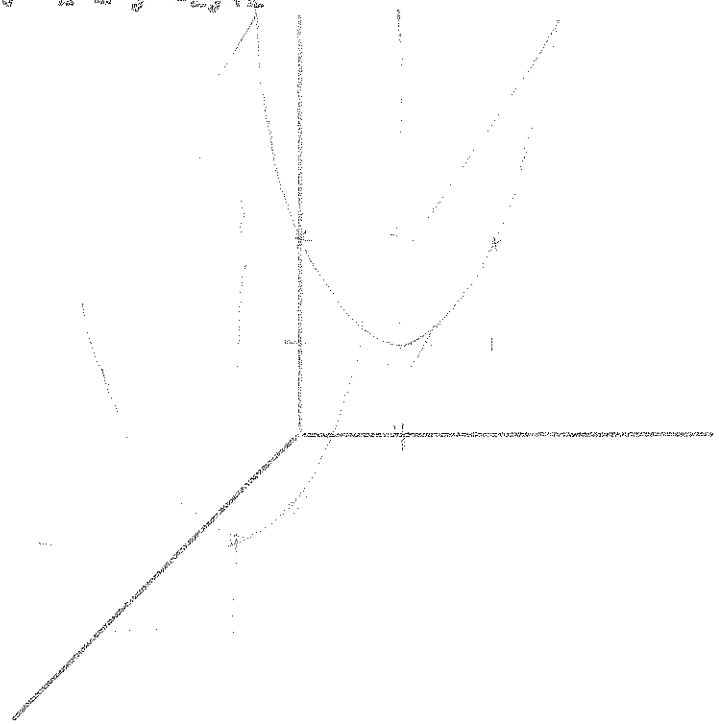
IV. Graph the following surfaces! 10 points each.

1. $z = y^2 - 2y + 2$

$z - 1 = y^2 - 2y + 1$

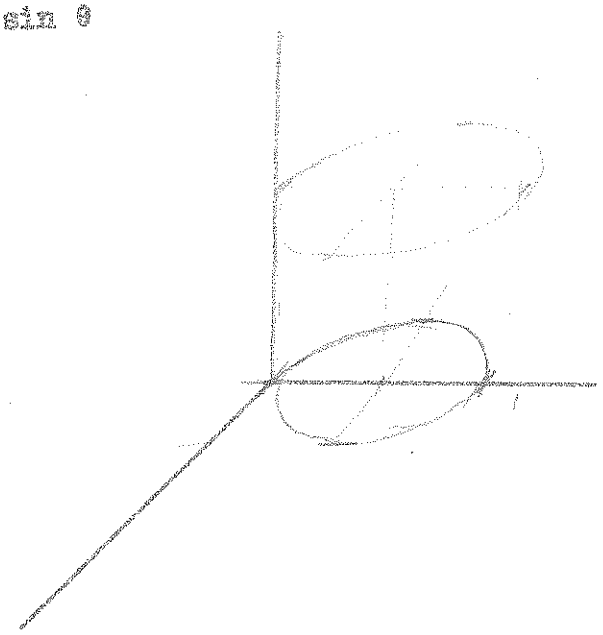
$z - 1 = (y - 1)^2$

parabolic cylinder



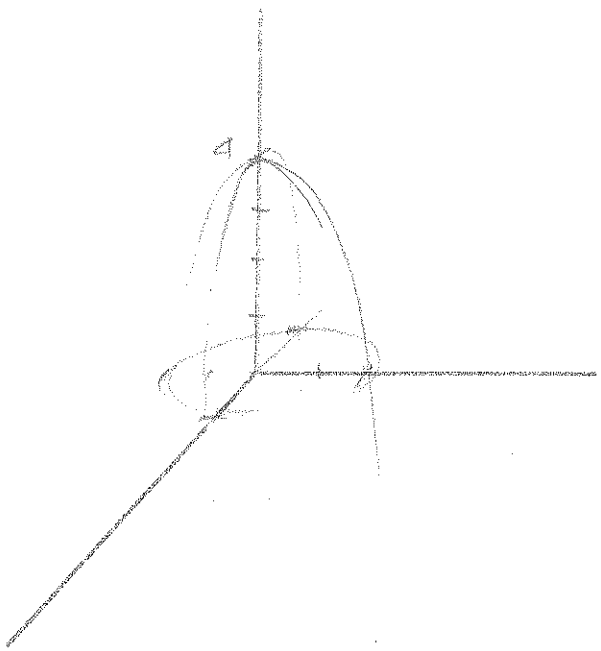
IV. cont.

2. $r = \sin\theta$



cylinder

3a. $z = 4 - 4x^2 - y^2$



$z = 4 - 4x^2$

$z = 4 - y^2$

$4x^2 + y^2 = 4$

$x^2 + \frac{y^2}{4} = 1$

elliptic paraboloid