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17
15

Math 232
Test I
April 18, 1975

Name KEY

Show work!

I. 6 points ea.

1. The intercepts of $x^2y + y^2 = 4$ are

$x=0 \quad y^2 = 4 \quad y = \pm 2 \quad \begin{pmatrix}(0,2)\\(0,-2)\end{pmatrix} \qquad y=0$ none

2. The symmetries of $x^2y^2 - xy^2 = 2$ are

x-axis

3. A rotation of axes will eliminate the xy-term in

$x^2 - 2\sqrt{3}\,xy - y^2 - 5 = 0$. What is the angle?

$\cot 2\alpha = \frac{A-C}{B} = \frac{1-(-1)}{-2\sqrt{3}} = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}} \qquad 2\alpha = \frac{2\pi}{3} \qquad \alpha = \frac{\pi}{3}$

4. At what angle do the curves $y = x^2+2$ and $y = 4-x^2$ meet?(or its tan.)

$x = +1$

$\frac{dy}{dx} = 2x \qquad \frac{dy}{dx} = -2x \qquad m_1 = 2 \quad m_2 = -2$

$x^2+2 = 4-x^2 \qquad \tan\beta = \frac{2-(-2)}{1+(2)(-2)} = \frac{4}{1-4} = -\frac{4}{3}$

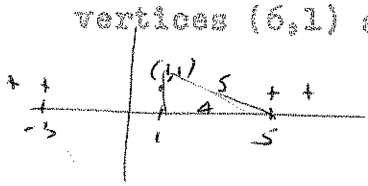
$2x^2 = 2 \quad x^2 = 1 \quad x = \pm 1$

or $\tan\beta = \frac{4}{3}$

5. The equation of the circle with center (1,-2) and radius 4 is

$(x-1)^2 + (y+2)^2 = 16$

6. The equation of the ellipse with foci at (5,1) and (-3,1), and vertices (6,1) and (-4,1) is



center (1,1)
$c = 4 \quad a = 5$
$b = 3$

$\frac{(x-1)^2}{25} + \frac{(y-1)^2}{9} = 1$

7. The equations of the asymptotes of the hyperbola $\frac{(x-1)^2}{4} - y^2 = 1$ are

slope $\pm\frac{b}{a} = \pm\frac{1}{2}$ center (1,0)

$\frac{y}{x-1} = \frac{1}{2} \qquad \frac{y}{x-1} = -\frac{1}{2}$

$y = \frac{x}{2} - \frac{1}{2} \qquad y = -\frac{x}{2} + \frac{1}{2}$

72

I. cont.

8. A rotation of $\alpha = \frac{\pi}{4}$ will eliminate the xy term in $xy = 4$. What will bw the equation in the new coordinates x', y' ?

$\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$

$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$

$\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) = 4$

$\frac{x'^2}{2} - \frac{y'^2}{2} = 4$

II. 10 points

What is the equation of the line normal to the curve $xy^2+2y = 3$ where it crosses the y-axis?

$x = 0 \quad 2y = 3$

$y = \frac{3}{2} \quad (0, \frac{3}{2})$

$x\,2y\frac{dy}{dx} + y^2 + 2\frac{dy}{dx} = 0$

$0 + \frac{9}{4} + 2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{9}{8}$

$m = \frac{8}{9}$

$\frac{y - 3/2}{x} = 8/9 \quad \boxed{y = 8/9\,x + \frac{3}{2}}$

III. 14 points ea. Do any THREE. 11:07

Carefully sketch the graph of the following curves on the graph paper. Identify by name if possible, draw asymptotes and vertices, and give symmetries.

1. $y^2-4y+4x = 0$

2. $16x^2+y^2+32x+8y+16 = 0$

3. $xy - x = 1$

4. $xy^2-2y^2 = 1$

$y^2 - 4y + 4 + 4x = 4$

$(y-2)^2 = -4x + 4$

$= -4(x-1)$

$x = 0$

$(y-2)^2 = 4$

$y - 2 = \pm 2$

2. $16(x^2 + \cancel{32}^{2}x + 1) + y^2 + 8y + 16 = \cancel{-16+16} + 16$

$16(x+1)^2 + (y+4)^2 = 16$

$(x+1)^2 + \frac{(y+4)^2}{16} = 1$

3. $xy - x - 1 = 0$

$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$

$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$

$\frac{x'^2}{2} - \frac{y'^2}{2} - \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}} = 1$

$x'^2 - y'^2 - \sqrt{2}x' = 2$

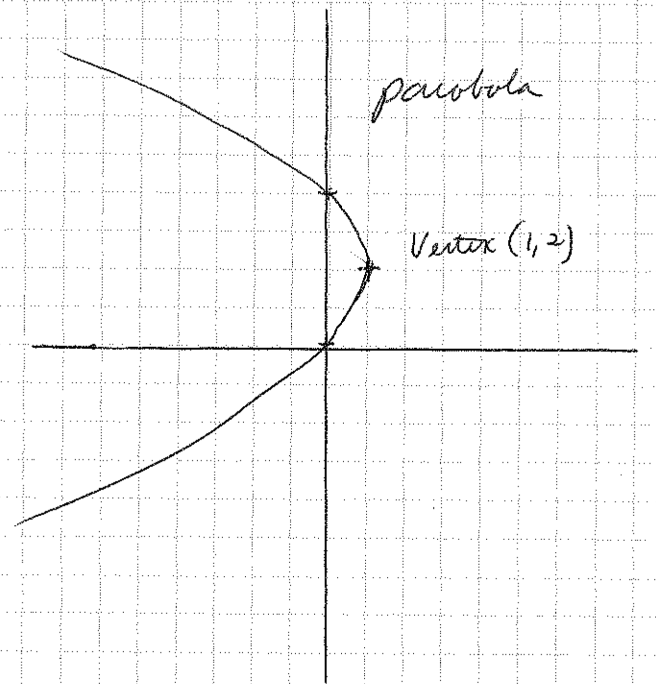
$(x'^2 - \sqrt{2}x' + \frac{1}{2}) - y'^2 = 2 + \frac{1}{2}$

$(x - \frac{\sqrt{2}}{2})^2 - y'^2 = \frac{5}{2}$

$\frac{(x - \frac{\sqrt{2}}{2})^2}{5/2} - \frac{y'^2}{5/2} = 1$

16

1.



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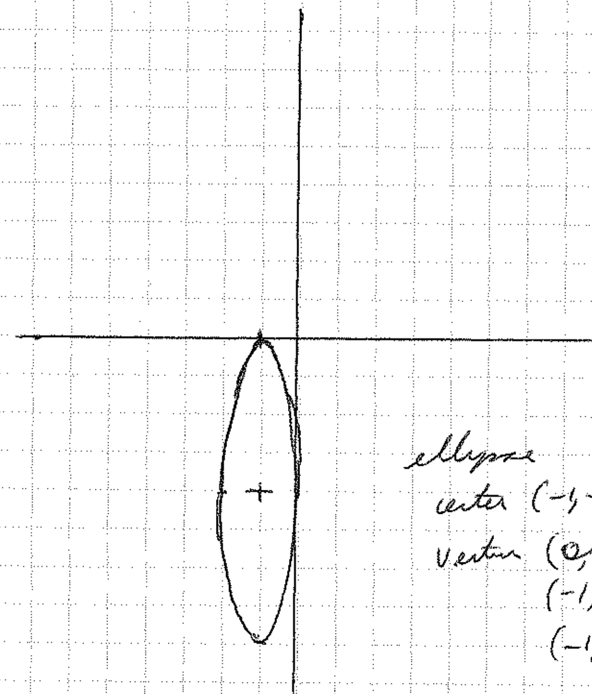
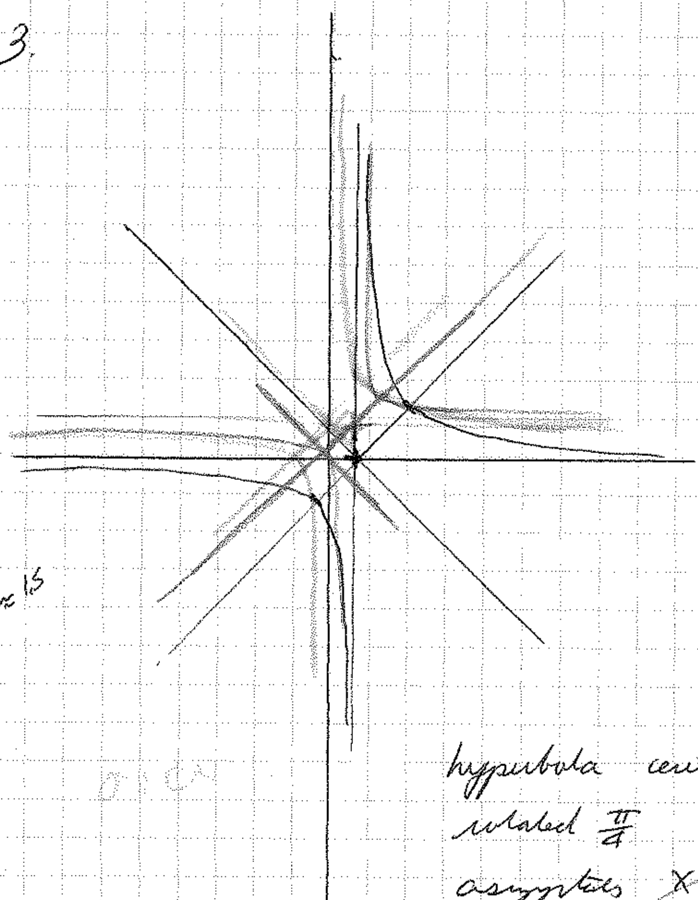


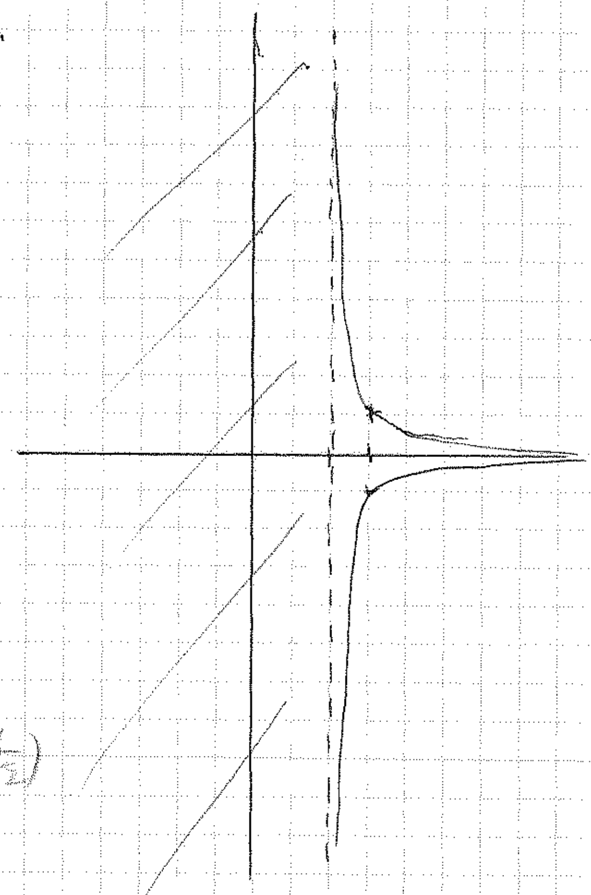
ellipse
center (−1, −4)
Vertex (0, −4)
(−1, 0)
(−1, −8)

3.



$\sqrt{5}/2 \approx 1.5$

4.



hyperbola center $\left(\frac{1}{\sqrt{2}}, \frac{0}{\sqrt{2}}\right)$
rotated $\frac{\pi}{4}$
asymptotes $\cancel{x = \frac{1}{\sqrt{2}}}$
$\cancel{y = 0}$
$y = 1, \ x = 1$