Math 232
Test II
May 9, 1975

Name KEY

Show work for full credit!

I. 5 points each.

1. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

2. $\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1} x + C$

3. $\cosh 0 = \frac{e^0 + e^0}{2} = 1$

4. What is the cartesian equation for the curve with parametric equations $x = \sinh t$, $y = \cosh t$?

$y^2 - x^2 = 1$

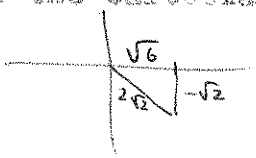
5. What are the cartesian coordinates of the point with polar coordinates $(2, \frac{2\pi}{3})$?

$x = 2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$ $\quad y = 2\sin\frac{2\pi}{3}$ $\quad y = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$ $\quad (-1, \sqrt{3})$

6. Find polar coordinates for the cartesian coordinates $(\sqrt{6}, -\sqrt{2})$.

$r^2 = 6 + 2 = 8$

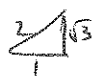
$r = 2\sqrt{2}$



$\tan\theta = \frac{-\sqrt{2}}{\sqrt{6}} = \frac{-1}{\sqrt{3}}$ $\quad \theta = -\frac{\pi}{6}$

$\left(2\sqrt{2}, -\frac{\pi}{6}\right)$

7. What is the slope of the tangent line(s) to the curve $r = \sin 3\theta$ at the pole?



$\sin 3\theta = 0$

$3\theta = 0, \pi, 2\pi, 3\pi$ $\quad \theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$ $\quad 0, \sqrt{3}, -\sqrt{3}$

8. What is the cartesian equation for the curve $r = 3\csc\theta$?

$r = \frac{3}{\sin\theta}$ $\quad r\sin\theta = 3$

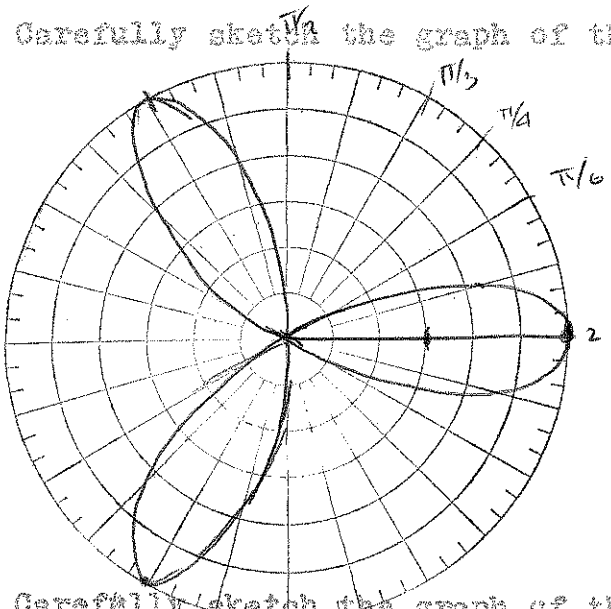
$y = 3$

80

04
16
12 min

II. 10 points each.

1. Carefully sketch the graph of the polar equation $r = 2\cos 3\theta$.



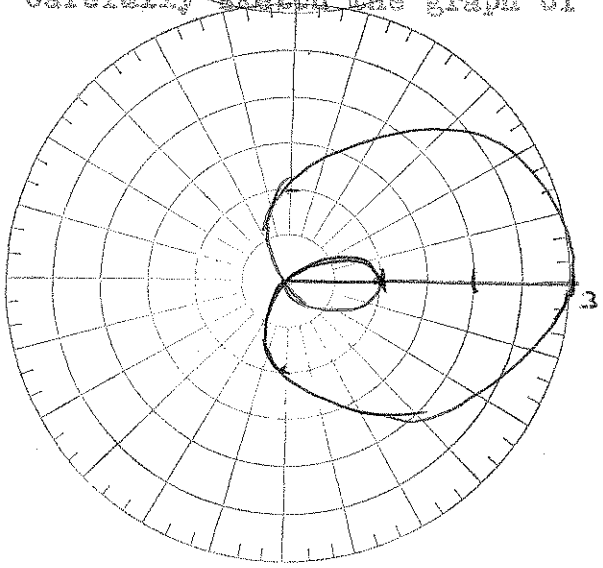
$3\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{6}$

$3\theta = \pi$

$3\theta = \frac{3\pi}{2}$

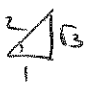
2. Carefully sketch the graph of the polar equation $r = 2\cos\theta - 1$.



$\cos\theta = 0$

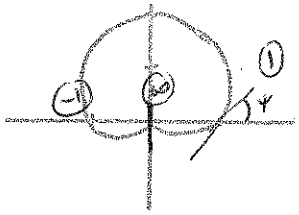
θ	r
0	1
$\frac{\pi}{2}$	−1
π	−3

$\cos\theta = \frac{1}{2}$



$\theta = \frac{\pi}{3}$

3. What is the slope of each of the lines tangent to the cardioid $r = 1 + \sin\theta$ at the x-intercepts?



$\frac{dr}{d\theta} = \cos\theta$

$\theta = 0 \quad \cos\theta = 1$

$\tan\psi_1 = \frac{1}{1} = 1$

$\psi_1 = \frac{\pi}{4}$

$\sin\theta = -1$

$\theta = \frac{3\pi}{2}$

III. 15 points each.

1. The parametric equations for the projectile example are
$x = 1600\cos\alpha\, t$, $y = -16t^2 + 1600\sin\alpha\, t$, $0 \le t \le 100\sin\alpha$.

a. What is the cartesian equation?

b. At what angle does the projectile hit the ground?

$$t = \frac{x}{1600\cos\alpha} \qquad y = -16\frac{x^2}{(1600)^2\cos^2\alpha} + \frac{1600\sin\alpha\, x}{1600\cos\alpha}$$

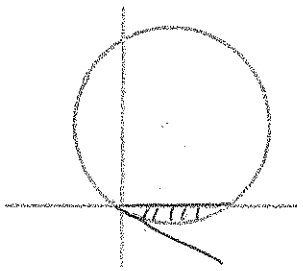
$t = 100\sin\alpha$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-32t + 1600\sin\alpha}{1600\sin\alpha}$$

$t = 100\sin\alpha$ $\qquad \frac{dy}{dx} = \frac{-3200\sin\alpha + 1600\sin\alpha}{1600\sin\alpha}$

$= -1$ $\qquad 45^\circ$

2. Find the area of the piece of the circle $r = \cos(\theta - \frac{\pi}{3})$ which lies below the polar axis.



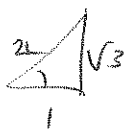
$\cos(\theta - \frac{\pi}{3}) = 0$

$\theta - \frac{\pi}{3} = \frac{\pi}{2}, -\frac{\pi}{2}$

$\theta = \frac{5\pi}{6}, -\frac{\pi}{6}$

$$\int_{-\pi/6}^{0} \frac{r^2}{2}\,d\theta = \int_{-\pi/6}^{0} \frac{\cos^2(\theta - \frac{\pi}{3})}{2}\,d\theta = \int_{-\pi/6}^{0} \frac{1 + \cos(2\theta - \frac{2\pi}{3})}{4}\,d\theta$$

$$= \left[\frac{\theta}{4} + \frac{\sin(2\theta - \frac{2\pi}{3})}{4}\right]_{-\pi/6}^{0} = \frac{\pi}{24} + \frac{\sin(-\frac{2\pi}{3})}{4} - \frac{\sin(-\pi)}{4}$$



$$= \frac{\pi}{12} + \frac{-\sqrt{3}/2}{4} = \frac{\pi}{24} - \frac{\sqrt{3}}{8}$$