Math 232
Test III
June 2, 1975

Name KEY

I. 5 points each.

1. The length of the vector $6\bar{i} - \bar{j}$ is

$\sqrt{36+1} = \sqrt{37}$

2. If $A = (1,2)$, $B = (2,3)$, and $C = (-2,4)$, then $2i+\bar{j} + (4i-j)$

$\overline{AB} + \overline{CB} = 5\ \bar{i} + 0\ \bar{j}$.

3. If $\bar{v}_1 = \bar{i} - 7\bar{j} + \bar{k}$ and $\bar{v}_2 = \bar{j} + 2\bar{k}$, then $-21+1$

$3\bar{v}_1 - \bar{v}_2 = 3\bar{i} - 22\bar{j} + \bar{k}$

4. If $\bar{R} = (t+1)\bar{i} + (3t^2-t)\bar{j}$, then $\frac{d\bar{R}}{dt} = \bar{i} + (6t-1)\bar{j}$

5. Find the unit tangent vector to the curve $x = e^t$, $y = 2t$ at $(1,2)$.

$x' = e^t \quad y' = 2$

$t =$

$\bar{T} = \frac{e^t}{\sqrt{e^{2t}+4}}\bar{i} + \frac{2}{\sqrt{e^{2t}+4}}\bar{j}$

6. What is the curvature of the curve $x = t^2-1$, $y = t+e^t$ at $t = 0$?

$x' = 2t \quad y' = 1+e^t$ $\qquad \cancel{t}\ x' = 0 \quad y' = 2$ $\qquad \pm\frac{0-4}{(\sqrt{4+4})^{3/2}} = \frac{1}{2}$

$x'' = 2 \quad y'' = e^t$ $\qquad x'' = 2 \quad y'' = 1$

7. If $\bar{A} = \bar{i}-\bar{j}+2\bar{k}$ and $\bar{B} = 2\bar{i}+2\bar{j}+7\bar{k}$, then $\bar{A}\cdot\bar{B} = 2 - 2 + 14 = 14$

8. (cont) $\bar{A}\times\bar{B} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 2 & 2 & 7 \end{vmatrix} \begin{matrix} i & j \\ 1 & -1 \\ 2 & 2 \end{matrix} = -7i + 4j + 2k + 2k - 4i - 7j = -11\bar{i} - 3\bar{j} + 4\bar{k}$

9. The normal vector to the plane $x+7y-z = 5$ is

$i + 7j - \bar{k}$

45

I. cont.

10. The vector projection of $\bar{i}+2\bar{j}-5\bar{k}$ onto $\bar{i}+\bar{j}$ is

$$\frac{\bar{A}\cdot\bar{B}}{\bar{A}\cdot\bar{A}}\cdot A = \frac{1+2+0}{2}\,\overline{i+j} = \frac{3}{2}i+\frac{3}{2}j$$

II. ~~15~~ 12 points each. OMIT ~~one~~ TWO. Do only ~~five~~ four:

1. A particle moves along the curve with its position at time t given by $x = t^2+1$, $y = t^3-t+2$, where t is in seconds and distance is in feet. Find the velocity, acceleration, and speed when the particle passes (5,8).

$$\bar{V} = 2t\bar{i} + (3t^2-1)\bar{j} \qquad \text{speed} = \sqrt{}$$

$$\bar{a} = 2\bar{i} + 6t\bar{j}$$

$$t = 2$$

$$\bar{V} = 4\bar{i} + 11\bar{j} \qquad \text{speed} = \sqrt{16+121} = \sqrt{137}$$

$$\bar{a} = 2\bar{i} + 12\bar{j}$$

2. What is the equation of the plane containing (1,3,5), (6,4,-2), and (2,-3,2)?

$$N = (5\bar{i}+\bar{j}+7k) \times (i-6j-3k)$$

$$= \begin{vmatrix} i & j & k \\ 5 & 1 & -7 \\ 1 & -6 & -3 \end{vmatrix} \begin{matrix} i & j \\ 5 & 1 \\ 1 & -6 \end{matrix} = -3i \; \bar{+}7j - 30k$$

$$-k \; \cancel{+} \; 42\bar{i} + 15j$$

$$= \cancel{39\bar{i} + 22j - 31k}$$

$$-45\bar{i} + 8\bar{j} - 31k$$

$-45x+8y-31z = D$

$-45+24-155 = D = \cancel{-12}\ -176$

-106
24

~~$39x+22y-31z = D$~~

$\boxed{-45x+8y-31z = -176}$

~~$39+66-155 = D = 104-155 = -51$~~ $39x+22y-31z = -51$

3. Find the equation(s)(any form) of the line which is perpendicular to the plane $2x-y+3z = 4$ and passes through (1,1,2).

$$N = 2\bar{i} - j + 3\bar{k}$$

~~$2x-y+3z = D$~~

$$\vec{R} = \bar{i}+j+2k+t(2i-j+3k)$$

$$= (2t+1)\bar{i} + (-t+1)j$$

$$+ (3t+2)k$$

25 or 35

II. cont.

4. A parallelogram is formed using (1,2,0), (2,7,2), and (3,5,-2) as vertices. a. Find a fourth vertex. b. Find the area.

$(i + 5j + 2k) \times (2i + 3j - 2k)$

$3,5$
$(3, 12, 4)$
$(4, 10, 0)$

$$\begin{vmatrix} i & j & k \\ 1 & 5 & 2 \\ 2 & 3 & -2 \end{vmatrix} \begin{matrix} i & j \\ 1 & 5 \\ 2 & 3 \end{matrix} = -10i + 4j + 3k - 10k - 6i + 2j$$

$$= -16i + 6j - 7k$$

$$|-16i + 6j - 7k| = \sqrt{256 + 36 + 49} = \sqrt{341}$$

$$\begin{array}{r} 256 \\ 36 \\ 49 \\ \hline 341 \end{array}$$

5. Find the equation of the line of intersection of the planes $x+y+z = 4$ and $x-y+z = 0$.

$(i + j + k) \times (i - j + k)$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} \begin{matrix} i & j \\ 1 & 1 \\ 1 & -1 \end{matrix} = i + j - k + i - j - k = 2i - 2k$$

$\cancel{x}=$
$y + z = 4$
$-y + z = 0$
$y = z$

$2y = 4$
$y = 2$
$(0,2,2)$

$\vec{R} = 2j + 2k + t(2i - 2k)$
$= 2ti + 2j + (-2t + 2)k$
$x = 2t \quad y = 2 \quad z = -2t + 2$

6. Find the equation of the plane tangent to the sphere $x^2+y^2+z^2=9$ at the point (2,2,1).

$2i + 2j + k$

$2x + 2y + z = D$

$D = 9$

$2x + 2y + z = 9$