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Tim OK

MATH 233

Final Exam

175

Name________________________

December 20, 1985

Double ☐ equal ☐ weight to a test

mch - 78%

Show work. Give solutions, not just answers.

1. State the definition for the limit $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$. (5)

2. Compute $\int_C x + y^2\, ds$, where C is the straight line starting at (-1,1) and ending at (3,5). (10)

$\langle x, y\rangle = \langle -1, 1\rangle + t\langle 4,4\rangle = \langle -1+4t, 1+4t\rangle$

$r' = \langle 4,4\rangle \quad \|r'\| = \sqrt{32} = 4\sqrt{2}$

$\int_0^1 (-1+4t) + (1+4t)^2 \sqrt{32}\,dt = \sqrt{32}\int_0^1 -1+4t+1+8t+16t^2\,dt$

$= \sqrt{32}\int_0^1 12t+16t^2\,dt = \sqrt{32}\;\; 6t^2 + \frac{16t^3}{3}\Big|_0^1 = \sqrt{32}\left(6+\frac{16}{3}\right) = \frac{34}{3}\sqrt{32}$

$= \frac{136}{3}\sqrt{2} = 64.111$

3. Is $(3x^2 + y^2)\,dx + (2xy + x^3 + \cos x)\,dy$ an exact differential? Justify. (5)

4. Use differentials to approximate $(1.01)\sin(-.01)$. (5)

$x \sin y \qquad dx = .01 \quad dy = -.01$

$x = 1,\ y = 0$

$dz = (\sin y)\,dx + (x\cos y)\,dy$

$= 0(.01) + dy = \boxed{-.01}$

$f(1,0) = 0$

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5. Find general solutions for each of these differental equations: (30)

a. $y' = x^2 / y$

b. $y' = (x^2 - y) / (x-y)$

c. $y' = (2x - 2y)/ (x+y)$

6. Compute ∇f for $f(x,y,z) = x^3y^2z - xy \sin z$. (5)

$$\nabla f = \langle 3x^2y^2z - y\sin z,\ 2y\,x^3z - x\sin z,\ x^3y^2 \mp xy\cos z \rangle$$

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(30)

7. Let $f(x,y) = \begin{cases} \sin y & , 0 < y < \pi \\ 0 & , \text{elsewhere.} \end{cases}$

a. Compute: $f(0,0)$, $f(\pi,0)$, $f(0,\pi)$.

b. For what points is this function discontinuous?

c. Sketch the graph, and give its name or type if it has one.

d. Find the equation of the plane which is tangent to this surface for $(x,y) = (\pi/2, \pi/4)$.

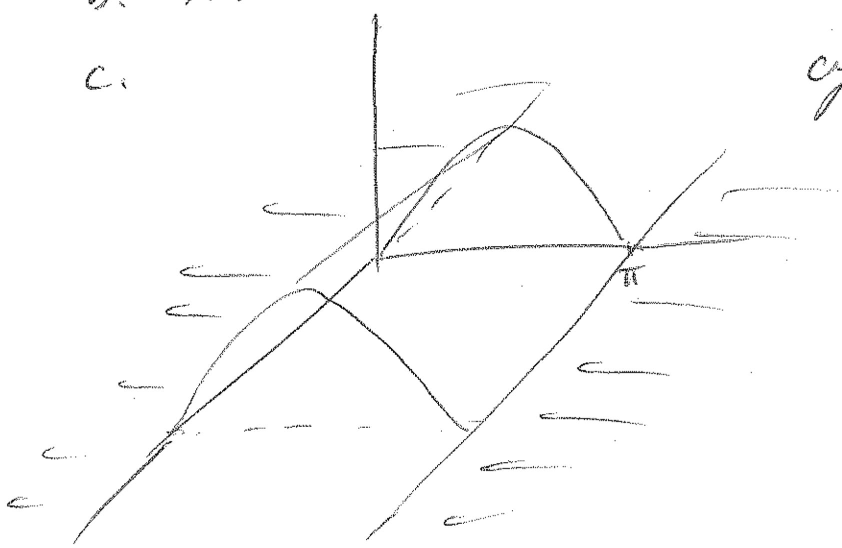
a. $f(0,0) = \sin 0 = 0$

$f(\pi,0) = \sin 0 = 0$

$f(0,\pi) = 0$

b. none

c.



cylinder

d. $F(x,y,z) = \sin y - z$

$\nabla F = \langle 0, \cos y, -1 \rangle$ $\quad \sin \frac{\pi}{4} = 1/\sqrt{2}$

at $(\pi/2, \pi/4)$ $\quad \nabla F = \langle 0, \cos \pi/4, -1 \rangle = \langle 0, 1/\sqrt{2}, -1 \rangle$

$0 + \frac{1}{\sqrt{2}}(y - \frac{\pi}{4}) - 1(z - 1/\sqrt{2}) = 0$

$\frac{\sqrt{2}}{2} y - z - \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} = 0$

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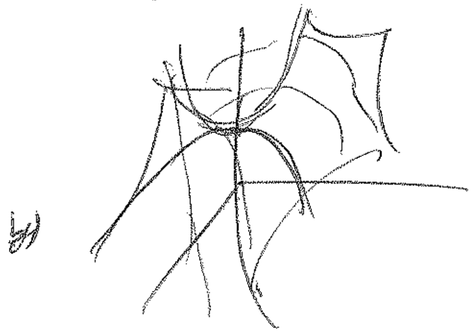
8. Consider the surface defined by $y^2 - x^2 + z = 1$. (20)

a. Sketch this surface. Give its name or type if it has one.

b. Find the point(s) on the surface for which the tangent plane is parallel to the plane $2x - 4y - z = 7$.



$z = x^2 - y^2 + 1$

b. $F(x,y,z) = y^2 - x^2 + z \cancel{=1} - 1$

$\nabla F = \cancel{2y - 2x}\ \langle -2x, 2y, +1 \rangle$

$\underline{u} = \langle 2, -4, -1 \rangle$

$\nabla F = \lambda \underline{u}$

$\langle -2x, 2y, 1 \rangle = \lambda \langle 2, -4, -1 \rangle$

$-2x = 2\lambda \quad 2y = -4\lambda \quad 1 = -\lambda$

$x = -\lambda = 1 \quad y = -2\lambda (= 2) \quad \lambda = -1$

$z = 1 - y^2 + x^2$
$= 1 - (2)^2 + (-1)^2$
$= 1 - 4 - 1 = -2$

$(1, 2, -2)$

9. Suppose that a bacteria population doubles in 30 minutes. If we start with a count of 100, how long will it take for the population to reach a count of 300? (10)

$f(t) = Ce^{kt}$

$= 100e^{kt}$

$2 = e^{k30}$

$k = \frac{1}{30}\ln 2$

$f(t) = \cancel{100}e^{\left(\frac{1}{30}\ln 2\right)t}$

$300 = 100\, e^{\left(\frac{1}{30}\ln 2\right)t}$

$\ln 3 = \frac{1}{30}\ln 2t$

$t = \frac{30\ln 3}{\ln 2} = \cancel{52.7}\ 47.5$

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10. Find the relative maxima and minima for $f(x,y) = \sin y - xy$. (10)

$f_x = -y \quad = 0 \qquad y = 0$

$f_y = \cos y - x \quad = 0 \qquad 1 - x = 0 \quad x = 1$

$(1, 0)$

$f_{xx} = 0$

$f_{yy} = -\sin y$

$f_{xy} = -1$

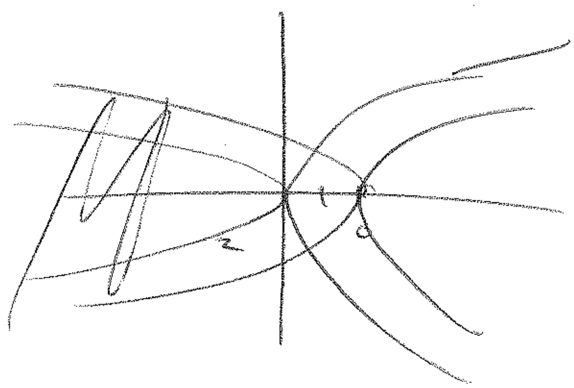
$M = 0 - (-1)^2 < 0$ saddle none

11. Draw two different level curves for $f(x,y) = y^2 - 3x + 2$. Label. (10)

$y^2 - 3x + 2 = 0$

$3x = y^2 + 2$

$3x = y^2 + 2 - a$



12. Let $z = x^2y + y^3$. At the point $(2,3)$ x is decreasing at the rate of 3 in./sec. and y is increasing at the rate of 2 in./sec. At what rate is z changing? Is z increasing or decreasing? (10)

$$\frac{dz}{dt} = 2xy\underbrace{\frac{dx}{dt}}_{=-3} + (x^2 + 3y^2)\underbrace{\frac{dy}{dt}}_{=2}$$

at $(2,3)$ $= 12(-3) + 31(2) = 26$

inc

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13. Compute the directional derivative of $f(x,y,z) = x^2 - xz^3 + 2y$ at the point $(1,2,4)$ in the direction from there towards the point $(2,3,2)$. (5)

$6\sqrt{6}$

14. Use double integrals to find the area of the region bounded by the curves $2x = -y^2$ and $x+9 = y^2$. (10)

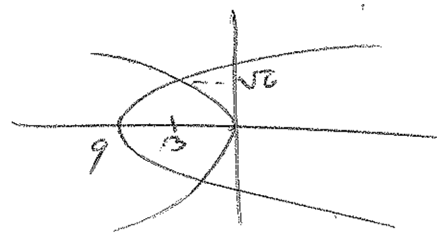
$x+9 = -2x$

$3x = -9$

$x = -3$

$y^2 = 6$

$y = \sqrt{6}$



$$2\int_0^{\sqrt{6}}\int_{y^2-9}^{-y^2/2} dx\,dy = 2\int_0^{\sqrt{6}} -\frac{y^2}{2} - y^2 + 9\,dy = 2\int_0^{\sqrt{6}} -\frac{3y^2}{2} + 9\,dy$$

$$= 2\left(-\frac{3y^3}{2} + 9y\right)\Big|_0^{\sqrt{6}} = \left(-\frac{18\sqrt{6}}{2} + 9\sqrt{6}\right)2 = \cancel{16\sqrt{6}}$$

$12\sqrt{6}$

29.39

15. Find the volume of the region in the first octant which is bounded by the coordinate planes and the surface $z = 9 - x^2 - y^2$. (10)

$$\int_0^{\pi/2}\int_0^3\int_0^{9-r^2} r\,dz\,dr\,d\theta = \int_0^{\pi/2}\int_0^3 9r - r^3\,dr\,d\theta$$

$$= \int_0^{\pi/2} \frac{9}{2}r^2 - \frac{r^4}{4}\Big|_0^3 = \int_0^{\pi/2} \left(\frac{81}{2} - \frac{81}{4}\right) d\theta$$

$$= \frac{\pi}{2}\left(\frac{81}{4}\right) = \frac{81\pi}{8}$$

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