MATH 233
TEST I
SEPTEMBER 27, 1985

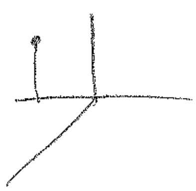
NAME KEY

Two 10 K
Fuel lift of 40 mi

median 80
HI 98
L 54
must move close

(5) 1. Give the definition of $\lim_{(x,y)\to(a,b)} f(x,y) = L$.

(5) 2. Find cylindrical and spherical coordinates for the point with rectangular coordinates (0,-1,1).

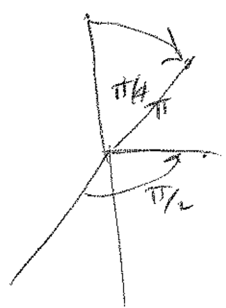


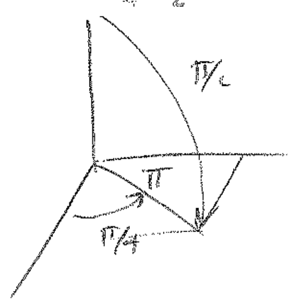
cyl $(1, -\frac{\pi}{2}, 1)$ or $(1, \frac{3\pi}{2}, 1)$

sph $(\sqrt{2}, -\frac{\pi}{2}, \pi/4)$ or $(\sqrt{2}, 3\pi/2, \pi/4)$

a lot missed θ

(5) 3. Convert spherical coordinates $(\pi, \frac{\pi}{4}, \frac{\pi}{2})$ to rectangular coordinates.





$(\sqrt{\pi/2}, \sqrt{\pi/2}, 1)$

$x = \sqrt{\pi}\ \frac{1}{\sqrt{2}}$

$y = \sqrt{\pi}/\sqrt{2}$

$z = 0$

$(\pi/\sqrt{2}, \pi/\sqrt{2}, 0)$

must get!

(15)

(21) 4. Find the following derivatives

a. $f(x,y) = ye^{xy} + x^2y$. Find $f_x(x,y)$.

$$f_x(x,y) = y^2e^{xy} + 2xy$$

b. $z = x^3y + x \sin y$. Find $\frac{\partial^2 z}{\partial x \partial y}$.

$$\frac{\partial z}{\partial y} = x^3 + x\cos y$$

$$\frac{\partial^2 z}{\partial x \partial y} = 3x^2 + \cos y$$

c. $f(x,y) = x \cos y$, $x = \sqrt{t}$, $y = t^3$. t. Find $\frac{df}{dt}$.

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= \cos y \frac{1}{2\sqrt{t}} + (-x\sin y)(3t^2 - 1)$$

(5) 5. A rectangle has a base which is increasing at 2 in./sec and height decreasing at the rate of 3 in./sec. When the base is 5 in. and the height is 4 in., is the area increasing or decreasing?

$$A = xy \qquad \frac{\partial A}{\partial x} = y \qquad \frac{\partial A}{\partial y} = x$$

$$\frac{dA}{dt} = \frac{\partial A}{\partial x}\frac{dx}{dt} + \frac{\partial A}{\partial y}\frac{dy}{dt}$$

$$= y\,2 + x(-3)$$

$$= 4\,2 + 5(-3) = 8 - 15 < 0 \qquad \text{dec}$$

$$-7 < 0$$

(10) 6. Evaluate (if limit exists)

a. $\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2} = \frac{1\cdot 1}{1+1} = \frac{1}{2}$

b. $\lim_{(x,y)\to(0,0)} g(f(x,y))$ where $f(x,y) = y\cos x$, and $g(u) = \cos u$.

$$f(90) = 0\cos 0 = 0$$

$$g(0) = \cos 0 = 1$$

(15)

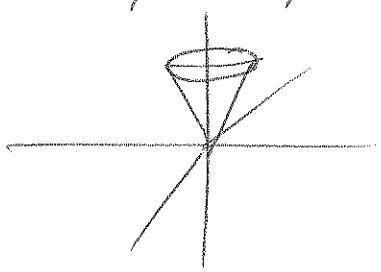
(5) 7. Explain why the following limit does not exist: $\lim_{(x,y) \to (0,0)} \frac{x^2}{x^2+y^2}$

x axis $y=0$ $= \frac{x^2}{x^2} = 1$

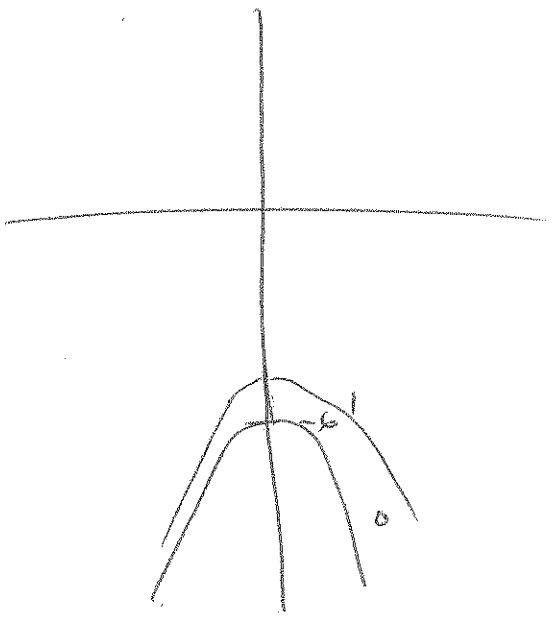
y axis $x=0$ $= 0$

(5) 8. Describe or sketch the graph of $\phi = \frac{\pi}{6}$ (in spherical coordinates)

cone, making angle $\pi/6$ with z axis



(5) 9. Sketch 2 level curves for $f(x,y) = 2x^2 + y + 6$ (give value).



$z = 2x^2+y+6 = 0$ 0

$y = -2x^2+6$

$2x^2+y+6 = 1$

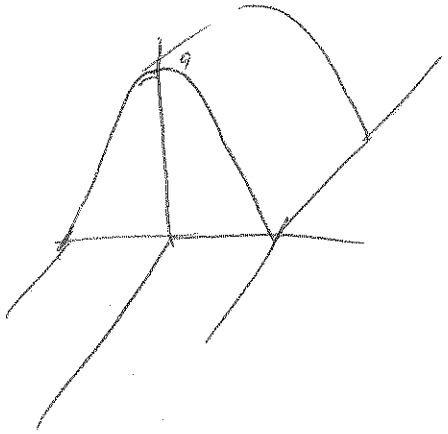
$y = -2x^2-5$

(14) 10. Let $f(x,y) = \begin{cases} 9 - y^2, & -3 < y < 3 \\ 0, & \text{elsewhere} \end{cases}$

a. $f(0,0) = 9$, $f(5,5) = 0$

b. Sketch the graph

c. For what points is this function continous?



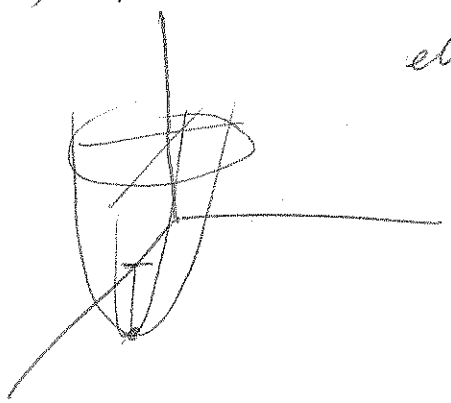
none.

(20) 11. Name and sketch the following surfaces.

a. $x^2 - 2x + y^2 = z$

$x^2 - 2x + 1 + y^2 = z + 1$

$(x-1)^2 + y^2 = z + 1$



elliptic paraboloid

b. $z^2 = x^2 + y^2$

$z^2 = x^2$ $z = \pm x$

$z = \pm y$

$x^2 + y^2 = k^2$

Cone