med = 81
$\bar{x} = 79$
(omit 10pts)
pages out of order.

MATH 233
Test III
November 25, 1985

Name KEY

Show work. Use methods studied in this course.

Very long
too much computation
But grades OK.
Some very good!

(10) 1. Find the equation of the plane which is tangent to the surface

$4 - x^2 - y^2 = z$ at the point (-1,2,-1).

$f(x) = 4 - x^2 - y^2 - z$

$\nabla f = \langle -2x, -2y, -1 \rangle$ at $(-1,2,-1)$ $\nabla f = \langle +2, -4, -1 \rangle$

most got.

$\langle +2, -4, -1 \rangle \cdot \langle x+1, y-2, z+1 \rangle = 0$

$+2(x+1) - 4(y-2) - (z+1) = 0$

$+2x - 4y - z + 2 + 8 - 1 = 0$ $\quad \boxed{+2x - 4y - z = -9 \text{ or } 2x + 4y + z = 9}$

1:3

(10) 2. At what point is the tangent plane to the graph of the function

$f(x,y) = xy - 2x$ parallel to the plane $x - 2y - 3z = 3$?

still lots of trouble. some forgetting $z = f(x,y)$. use T or something else for z.

$N = \langle 1, -2, -3 \rangle$

$\nabla F = (xy - 2x - z$

$\nabla F = \langle y-2, x, -1 \rangle$

$\langle y-2, x, -1 \rangle = \lambda \langle 1, -2, -3 \rangle$

$-1 = \lambda(-3) \quad \lambda = +\frac{1}{3}$

$y - 2 = \lambda$

$x = -2\lambda$

$x = -2/3$

$y - 2 = +1/3$

$y = 2\frac{1}{3} = 7/3$

$\boxed{\left(-2/3, 7/3, -4/3\ -2/9\right)}$

$z = -\frac{2}{3}\left(\frac{7}{3}\right) + \frac{4}{3}$

$= -\frac{14}{9} + \frac{12}{9} = -\frac{2}{9}$

3:30

(16) 3. Evaluate the following integrals:

a. $\int_0^2 \int_0^{x^2} \int_0^{xy} 3xy \, dz\,dy\,dx$

$$\int_0^2 \int_0^{x^2} 3xy(xy)\,dy\,dx = \int_0^2 \int_0^{x^2} 3x^2y^2\,dy\,dx$$

$$= \int_0^2 \frac{\cancel{3}x^2y^3}{\cancel{3}}\Big|_0^{x^2} dx = \int_0^2 x^8\,dx = \frac{x^9}{9}\Big|_0^2 = \boxed{\frac{2^9}{9}} \checkmark \quad \boxed{\frac{512}{9}} \checkmark$$

4:30

b. $\int_0^{2\pi} \int_0^2 \int_0^r \theta \, dz\,dr\,d\theta$

$$\int_0^{2\pi} \int_0^2 r\theta\,dr\,d\theta = \int_0^{2\pi} \frac{r^2\theta}{2}\Big|_0^2 d\theta = \int_0^{2\pi} 2\theta\,d\theta = \theta^2\Big|_0^{2\pi} = \boxed{(2\pi)^2}$$

/36

5:15

8. $\int_C x + y \, ds$, where C is the straight line beginning at the point

7 (1,2) and ending at the point (5,1).

$r(t) = (1,2) + t(4,-1)$

$= \langle 1+4t, 2-t \rangle$

$r'(t) = \langle 4, -1 \rangle \qquad |r'(t)| = \sqrt{17}$

$\int_C x+y \, ds = \int_0^1 (1+4t) + (2-t)\sqrt{17}\, dt$

$= \int_0^1 (3+3t)\sqrt{17}\, dt$

$= \left(3t + \frac{3}{2}t^2\right)\sqrt{17}\Big|_0^1$

$= \left(3 + \frac{3}{2}\right)\sqrt{17} = \frac{9\sqrt{17}}{2}$

6

(9) Find the work done by the force $\vec{F}(x,y) = 2x\,\vec{i} + y^2\,\vec{j}$ moving an object along the curve $\vec{r}(t) = 3t\,\vec{i} + t^3\,\vec{j}$. $1 \le t \le 3$

$r' = \langle 3, 3t^2 \rangle$

$W = \int_C F \cdot r'\, dt = \int_1^3 \langle 2(3t), t^6 \rangle \langle 3, 3t^2 \rangle\, dt$

$= \int_1^3 18t + 3t^8\, dt = \frac{18t^2}{2} + \frac{3t^9}{9}\Big|_1^3 = 9(3)^2 + \frac{1}{3}3^9 - \left[9 + \frac{1}{3}\right]$

$= 81 + 3^8 - 9 - \frac{1}{3} = 72 + 3^8 - \frac{1}{3} = 2\cancel{238}\frac{2}{3}$ $\qquad \frac{19898}{3}$

8:30

(14) 8. Find the mass and the center of mass of the lamina defined by the region bounded by the given curves curves with the given density ρ.

$x = y^2$, $y = x - 2$, $\rho(x,y) = 2$.

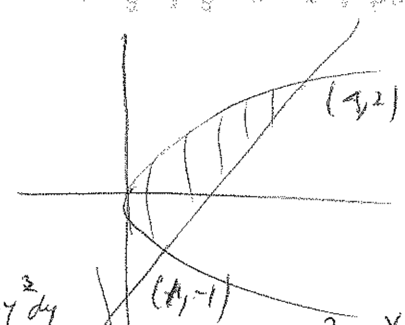
$21 - 6\frac{3}{5}$ $\quad \frac{105-33}{5} = \frac{72}{5}$

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0 \qquad y = -1, 2$

$x = 1, 4$



$M = \frac{9}{\cancel{4}}$ $\qquad \bar{x} = \frac{72/5}{9} = \frac{8}{5} \qquad \bar{y} = \frac{9/2}{9} = \frac{1}{2}$ $\qquad \left(\cancel{\frac{9}{4}}, \cancel{\frac{3}{2}}\right)$ $\quad \frac{17}{16}$

$\left(\frac{8}{5}, \frac{1}{2}\right)$

$\int_{-1}^{2} 2y^2 + 2y - 2y^3\, dy$

$\frac{2y^3}{3} + y^2 - \frac{2y^4}{4}\Big|_{-1}^{2}$

$= \frac{16}{3} + 4 - 8$

$-\left(-\frac{2}{3} + 1 - \frac{1}{2}\right)$

$= \frac{16}{3} - 4 + \frac{2}{3} - 1 + \frac{1}{2}$

$= 6 - 4 - 1 + \frac{1}{2}$

$= 3\frac{1}{2}\ 1/2$

$\text{mass} = \int_{-1}^{2}\int_{y^2}^{y+2} 2\, dx\, dy = 2\int_{-1}^{2} y+2-y^2\, dy = 2\left(\frac{y^2}{2} + 2y - \frac{y^3}{3}\right)\Big|_{-1}^{2}$

$= 2\left(2 + 4 - \frac{8}{3} - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right)$

$= 2\left(\cancel{8} - \frac{1}{2} - \frac{9}{3}\right) = \cancel{2\left(4 - \frac{3}{6} - \frac{4}{6}\right)} = \cancel{\frac{17}{6}}$

9 $\quad \frac{9}{2}$

$M_{x=0} = \int_{-1}^{2}\int_{y^2}^{y+2} x\, 2\, dx\, dy \qquad \int_{-1}^{2} x^2\Big|_{y^2}^{y+2} dy = \int_{-1}^{2} (y+2)^2 - y^4\, dy = \frac{(y+2)^3}{3} - \frac{y^5}{5}\Big|_{-1}^{2}$

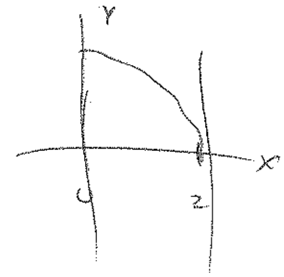
$= \frac{4^3}{3} - \frac{2^5}{5} - \left(\frac{1}{3} + \frac{1}{5}\right) = \frac{63}{3} - \frac{33}{5} = 21 - 6\frac{3}{5}$

12 15:30

$M_{y=0} \int_{-1}^{2}\int_{y^2}^{y+2} y\, 2\, dx\, dy = \int_{-1}^{2} 2y(y+2-y^2)\, dy$

4. Change to cylindrical coordinates and evaluate :

8 $\int_1^3 \int_0^2 \int_0^{\sqrt{4-x^2}} 3\, dy\, dx\, dz = \cancel{\int_1^3 \int_0^2 \int}$

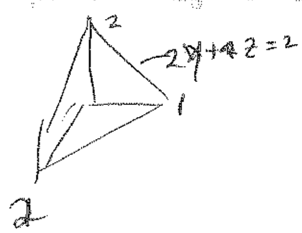


$$= \int_1^3 \int_0^{\pi/2} \int_0^2 3r\, dr\, d\theta\, dz$$

$$= \int_1^3 \int_0^{\pi/2} \frac{3r^2}{2}\Big|_0^2 = \int_1^3 \int_0^{\pi/2} 6\, d\theta\, dz = \int_1^3 3\pi\, dz = 6\pi$$

(20) 5. Find the volume of the region bounded by each of the following:

a. the plane $x + 2y + z = 2$, and the coordinate planes.



$$\int_0^2 \int_0^{1-z/2} \int_0^{2-2y-z} dx\, dy\, dz = \int_0^2 \int_0^{1-z/2} 2-\cancel{x}-2z\; dy\, dz$$

$\frac{2}{3}$

$$= \cancel{\int_0^2 (2-x)y - y^2 \Big|_0^{1-z/2} dz} = \int_0^2 (2-2z)y - \frac{y^2}{2}\Big|_0^{1-z/2} dz$$

$$= \int_0^1 (2-2z)\left(1-\frac{z}{2}\right) - \frac{\left(1-\frac{z}{2}\right)^2}{2} dz$$

$$= \int_0^1 2-3z+z^2 - \frac{\left(1-z+\frac{z^2}{4}\right)}{2} dz$$

b. the cylinder $x^2 + y^2 = 9$, and the planes $z = 0$, and $z = x + 2y$.

$$\int_0^{2\pi} \int_0^3 \int_0^{r\cos\theta+2r\sin\theta} r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^3 r(r\cos\theta + 2r\sin\theta)\, dr\, d\theta$$

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$$= \int_0^{2\pi} \int_0^3 r^2\cos\theta + 2r^2\sin\theta\, dr\, d\theta = \int_0^{2\pi} \frac{r^3}{3}\cos\theta + \frac{2r^3}{3}\sin\theta \Big|_0^3 d\theta$$

$$= \int_0^{2\pi} 9\cos\theta + 18\sin\theta\, d\theta = 9$$

~~(14)~~ 6. Evaluate the following line integrals:

(7) a. $\int_C x^2 - y\, dx$, where C is the curve $\vec{r}(t) = 2t^2\,\vec{i} + (t+1)\,\vec{j} + 2\vec{k}$. $1 \le t \le 3$

$$\int_1^3 \left((2t^2)^2 - (t+1)\right) 4t\, dt = \int_1^3 (4t^4 - t - 1)(4t) = \int_1^3 16t^5 - 4t^2 - 4t\, dt$$

$$= \frac{16t^6}{6} - \frac{4t^3}{3} - 2t^2\Big|_1^3 = \frac{16\cdot 3^6}{6} - 36 - 18 - \left(\frac{16}{6} - \frac{4}{3} - 2\right)$$

$= 8\cdot 3^5 - 36 - 18$
$- \frac{8}{3} + \frac{4}{3} + 2 = \boxed{-\frac{4}{3} - 56 - 8\cdot 3^5}$ $\frac{5672}{3}$ =

$= \cancel{8\cdot 24 - 56 + \frac{4}{3}}$