Yen

Math 233
Test I
October 23, 1975

Name KEY

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Part I In class.

7 min

1. Let $z = x^2y + 3x - 2xy^3$.
   a. Find the equation of the plane tangent to this surface at (2,1,7).
   b. Find the equations(s) of the normal line at this point.
   c. Find Δz_{tan} at (2,1) with $\Delta x = .1$ and $\Delta y = -.2$.
   d. Find the directional derivative $\frac{dz}{ds}$ in the direction $\bar{i} + \bar{j}$ at (1,1).

a. $\frac{\partial z}{\partial x} = 2x + 3 - 2y^3$ at (2,1,7) $4 + 3 - 2 = 5$

$\frac{\partial z}{\partial y} = x^2 - 6xy^2$ $\quad 4 - 12 = -8$

$N = 5\bar{i} - 8\bar{j} - \bar{k}$

a. $5(x-2) - 8(y-1) - (z-7) = 0$ or $5x - 8y - z = -5$

b. $\frac{x-2}{5} = \frac{y-1}{-8} = \frac{z-7}{-1}$

c. $\Delta z_{tan} = 5\Delta x - 8\Delta y$

$= 5(.1) - 8(-.2) = .5 + 1.6 = \cancel{1.1}$ 2.1

d. $\frac{dz}{ds} = 5\cos\phi - 8\sin\phi$

$= \cancel{5\frac{1}{\sqrt{2}} - \frac{8}{\sqrt{2}}} = -\frac{3}{\sqrt{2}}$

at (1,1) $\frac{\partial z}{\partial x} = 2 + 3 - 2 = 3$

$\frac{\partial z}{\partial y} = 1 - 6 = -5$

$\frac{dz}{ds} = 3\left(\frac{1}{\sqrt{2}}\right) - 5\left(\frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

Part I

2. Approximate $(1.01)^3(3.03) + 2(1.01)(4.97)^2$. $\approx 53,\cancel{57}$ $.02$

$1 \cdot 3 + 2 \cdot 1 \cdot 5^2 = 53$

$w = x^3 y + 2xz^2$

$\frac{\partial w}{\partial x} = 3x^2y + 2z^2$ $\quad \frac{\partial w}{\partial y} = x^3$ $\quad \frac{\partial w}{\partial z} = 4xz$

at 1, 3, 5 $\quad = 1 \quad = 20$

$= 9 + 50 = 59$

$dw = 59\,dx + dy + 20\,dz = 59(.01) + 1(.03) + 20(-.03)$

3. If $f(x,y) = x \cos y + x^2 y$, find $\frac{\partial^2 f}{\partial x \partial y}$. $\quad = .59 + .03 - .60 = .59$ $.02$

$\frac{\partial f}{\partial y} = -x \sin y + x^2$

$\frac{\partial^2 f}{\partial x \partial y} = -\sin y + 2x$

4. What is the direction of maximum increase for the function $w = x^2y + x^3y^3 - 6z$ at $(1,1,1)$?

$\frac{\partial w}{\partial x} = 2xy + 3x^2y^3$

$\nabla w = 5i + 4j - 6k$

$\frac{\partial w}{\partial y} = x^2 + 3x^3y^2$

$\frac{\partial w}{\partial z} = -6$

5. Find ∇f for the function $w = x \sin y - y^2 \cos z$ at $(2,0,0)$.

∇f $\quad \frac{\partial w}{\partial x} = \sin y$ $\quad \frac{\partial w}{\partial y} = x \cos y - 2y \cos z$

$\frac{\partial w}{\partial z} = +y^2 \sin z$

$\nabla f = 0i + 2j + 0k$

$= 2j$

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9 ea outside
12 inside

Part II

1. In what direction is the directional derivative of the function $w = 2x^2y + xy^3 - 2$ at $(2,1)$ equal to 0 ?

$$\frac{dw}{ds} = \nabla w \cdot \bar{u}$$

$$\nabla w = 9i + 14j$$

$$\bar{u} = \frac{-14i + 9j}{\sqrt{9^2+14^2}}$$

$$\frac{\partial w}{\partial x} = 4xy + y^3$$

$$\frac{\partial w}{\partial y} = 2x^2 + 3xy^2$$

$(2,1)$

$$\frac{\partial w}{\partial x} = 8 + 1 = 9$$

$$\frac{\partial w}{\partial y} = 8 + 6 = 14$$

OR

$$\frac{dw}{ds} = 9\cos\phi + 14\sin\phi = 0$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = -\frac{9}{14}$$

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Part II

2. At what points on the surface $x^3 + yz = 0$ is the tangent plane parallel to the xz plane?

$w = x^3 + yz$

$\nabla w = 3x^2 \bar{i} + z\bar{j} + y\,k$

$\vec{N} = 3x^2 \bar{i} + z\bar{j} + y\bar{k}$

$= C\bar{j}$

$3x^2 = 0 \quad x = 0$

$y = 0 \quad y = 0$

$z =$ anything

$\boxed{z \text{ axis}}$

Part II

3. Is $(x^2+y^2)dx + (2xy-2)dy$ the exact differential for some function? If so find it.

$$\frac{\partial M_x}{\partial y} = \frac{\partial N}{\partial x} \qquad \frac{\partial M}{\partial y} = 2y \qquad \frac{\partial M}{\partial x} = 2y \quad \checkmark$$

$$f(x,y) = \int M\,dx = \frac{x^3}{3} + xy^2 + C(y)$$

$$2xy-2 = \frac{\partial f}{\partial y} = 2xy + C'(y)$$

$$C'(y) = -2$$

$$C(y) = -2y + c$$

$$f(x,y) = \frac{x^3}{3} + xy^2 - 2y + C$$

OR

$$f(x,y) = \int N\,dy = \cancel{2}xy^2 - 2y + C(x)$$

$$\frac{\partial f}{\partial x} = \cancel{2xy - 2 + C} \; y^2 + C'(x)$$

$$\shortparallel$$

$$x^2 + y^2$$

$$C'(x) = x^2$$

$$C(x) = \frac{x^3}{3} + C$$

$$f(x,y) = \cancel{y^2 + \frac{x^3}{3}}$$

$$xy^2 - 2y + \frac{x^3}{3} + C$$

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Part II.

4. Find the least squares line y = mx +b for the points:

x	-2	-1	0	1	2
y	2	3	2	4	3
	-4	-3	0	4	6

$\sum x_i = 0$ $\qquad n = 5$

$\sum x_i^2 = 10$

$\sum y_i = 14$

$\sum x_i y_i = 3$

$$\begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ 14 \end{bmatrix}$$

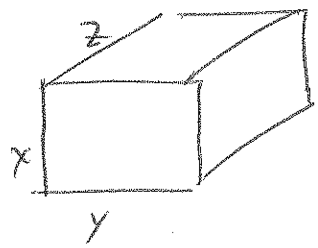
$10m = 3 \qquad m = .3$

$5b = 14 \qquad b = \frac{14}{5} = 2.8$

$$y = .3x + 2.8$$

Part II.

5. The size of a package which can be sent by parcel post or some other means is often limited by length plus girth. What are the dimensions of the largest rectangular box when this limit is 100 inches?



$$2x + 2y + z = 100$$

$$z = 100 - 2x - 2y$$

$$V = xyz$$

$$V = xy(100 - 2x - 2y)$$

$$= 100xy - 2x^2y - 2xy^2$$

$$\frac{dV}{dx} = 100y - 4xy - 2y^2 = 0$$

$$\frac{dV}{dy} = 100x - 2x^2 - 4xy = 0$$

$$100y - 2y^2 = 100x - 2x^2$$

$$y = \frac{100x - 2x^2}{4x} = 25 - \frac{x}{2}$$

$$100\left(25 - \frac{x}{2}\right) - 4x\left(25 - \frac{x}{2}\right) - 2\left(25 - \frac{x}{2}\right)^2 =$$

$$\left(25 - \frac{x}{2}\right)\left[100 - 4x - 50 + x\right] = 0$$

$$\left(25 - \frac{x}{2}\right)(50 - 3x) = 0$$

$$x = \frac{50}{3} \qquad x = 50$$

$$\times \quad V = 0$$

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$$y = 25 - \frac{25}{3}$$

$$= \frac{50}{3}$$

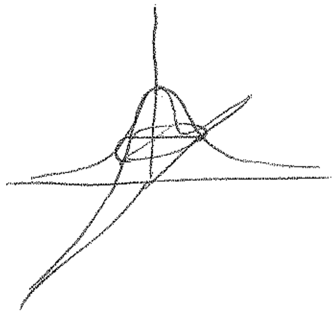
$$z = 100 - \frac{100}{3} - \frac{100}{3}$$

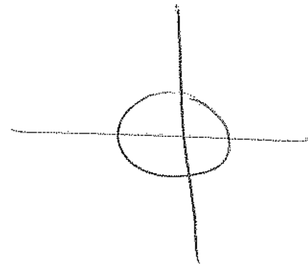
$$= \frac{100}{}$$

$$\boxed{\frac{50}{3} \times \frac{50}{3} \times \frac{100}{3}}$$

Part II.

6. Carefully sketch the graph of $z = e^{-x^2-y^2}$. Also sketch one of the level curves.





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