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43
15
watch int
(set ups only)

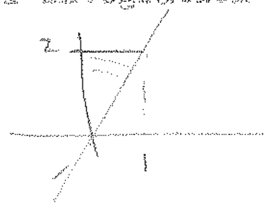
Math 233
Test II
Nov. 25, 1975 Name KEY

alt int setup only

One sheet of notes allowed.

10 1. Reverse the order of integration in the following iterated integral.

$$\int_0^2 \int_0^{y/2} dx\, dy$$



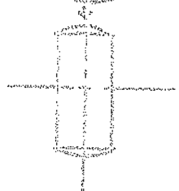
$x = y/2 \quad y = 2x$

$$\int_0^1 \int_{2x}^{2} dy\, dx$$

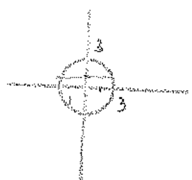
15 2. Set up only! Find the volume of the region inside the sphere centered at (0,0,0) with radius 5 which is also within the cylinder $x^2+y^2 = 9$. a. cartesion coordinates. b. ~~polar~~ or cylindrical coordinates.

$x^2+y^2+z^2=25$

Side view



Top view



$$\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{-\sqrt{25-x^2-y^2}}^{\sqrt{25-x^2-y^2}} dz\, dx\, dy$$

or $$\int_{-5}^{5}\int\int^{x} dx\, dy\, dz$$

$$\int_0^{2\pi}\int_0^{3}\int_{-\sqrt{25-r^2}}^{\sqrt{25-r^2}} r\, dz\, \cancel{r}\, dr\, d\theta$$

$z = \sqrt{25-r^2}$

OR $$2\int_0^{2\pi}\int_0^{3}\sqrt{25-r^2}\; r\, dr\, d\theta$$

10 3. Find the polar moment of inertia I_0 of the disk centered at (0,0) with radius 3. (Assume uniform density.)

SET UP

$$I_0 = \iint r^2\, dA = \int_0^{2\pi}\int_0^{3} r^3\, dr\, d\theta = \int_0^{2\pi} \frac{r^4}{4}\Big|_0^3 d\theta = \int_0^{2\pi} \frac{3^4}{4}\Big|\, d\theta$$

$$= 2\pi \cdot \frac{3^4}{4} = \frac{3^4\pi}{2}$$

OR $$\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} x^2+y^2\, dx\, dy = \int_{-3}^{3} \frac{x^3}{3} + y^2 x \Big|_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} dy$$

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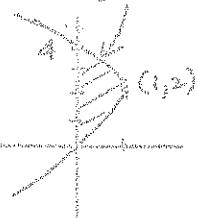
15 4. Find the x-coordinate, $\bar{x}$, of the center of mass of the region: $-4(x-1) = (y-2)^2$

SET UP

Extra credit: Find $\bar{y}$. $\quad -4x = (y-2)^2 - 4 \qquad x = 1 - \frac{(y-2)^2}{4}$



$$M_y = \iint y \, dA = \int_0^4 \int_0^{1-\frac{(y-2)^2}{4}} \cancel{x}\, dx\, dy \qquad -4x = y^2 - 4y \qquad x = -\frac{y^2}{4} + y$$

$$m = \int_0^4 \int_0^{1-\frac{(y-2)^2}{4}} dx\, dy$$

$$= \int_0^4 yx \Big|_0^{1-\frac{(y-2)^2}{4}} dy = \int_0^4 y - y\frac{(y-2)^2}{4}\, dy \qquad y^2 - 4y + 4$$

$$= \int_0^4 1 - \frac{(y-2)^2}{4}\, dy$$

$$= \int_0^4 y - \frac{y^3}{4} + y^2 - \cancel{y}\, dy = \frac{y^4}{16} + \frac{y^3}{3}\Big|_0^4$$

$$= y - \frac{(y-2)^3}{12}\Big|_0^4$$

$$= \frac{4^4}{16} + \frac{4^3}{3} = 16 + \frac{64}{3} = \frac{112}{3}$$

$$= 4 - \frac{24}{16} + \frac{24}{16} = 4 \qquad \bar{x} = \frac{\frac{112}{3}}{4} = \frac{28}{3}$$

10 5. Write as a fraction: $.1321212121\ldots = .13\overline{21}$

$$= \frac{13}{100} + \frac{21}{100^2} + \frac{21}{100^3} + \cdots = \frac{13}{100} + \frac{21}{100^2}\sum_{n=0}^{\infty}\frac{1}{100^n} = \frac{13}{100} + \frac{21}{100^2}\frac{1}{1-\frac{1}{100}}$$

$$= \frac{13}{100} + \frac{21}{100}\frac{1}{\frac{99}{33}} = \frac{429+7}{33\cdot 100} = \frac{436}{3300} = \frac{109}{33\cdot 25} = \frac{109}{825}$$

13
33
39
39
21
429

25
33
75
75
825

9400

436
3
1308

10 6. Write out the first 5 terms of the Maclaurin series for $\ln(x+1)$. Write the power series in sigma notation.

at $x=0$

$y = \ln(x+1)$ $\qquad 0$

$y' = \frac{1}{x+1} = (x+1)^{-1}$ $\qquad 1$

$y'' = -(x+1)^{-2}$ $\qquad -1$

$y''' = 2(x+1)^{-3}$ $\qquad 2$

$y^{iv} = -6(x+1)^{-4}$ $\qquad -6$

$$x - \frac{x^2}{2} + \frac{2x^3}{6} - \frac{6x^4}{24}$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

$$\sum_{n=1}^{\infty} (-1)^{n+1}\frac{x^n}{n}$$

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30 7. Determine whether each series converges or not. Give reasons.

a. $\sum_{k=1}^{\infty} \frac{2}{3^k}$ b. $\sum_{k=1}^{\infty} \frac{3}{k^3}$ c. $\sum_{k=2}^{\infty} \frac{k+1}{2k}$ d. $\sum_{k=1}^{\infty} \frac{k}{k^5+10}$

e. $\sum_{k=1}^{\infty} \frac{k^2+1}{k!}$

a. $2\sum\left(\frac{1}{3}\right)^k$ Geometric

$r = \frac{1}{3} < 1$

Converges

b. $p = 3$ series $p > 1$ converges

c. $\frac{k+1}{2k} \to \frac{1}{2} \neq 0$ div

d. $\frac{k}{k^5+10} \le \frac{k}{k^5} = \frac{1}{k^4}$

$\sum \frac{1}{k^4}$ conv $\Rightarrow$ conv

CT.

e. k^2+2k+2

$$\frac{\frac{(k+1)^2+1}{(k+1)!}}{\frac{k^2+1}{k!}} = \frac{(k+1)^2+1}{(k^2+1)(k+1)} \to 0 < 1$$

k^3+k+k^2+1 RT.

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