Math 233
Test I
April 20, 1978

Name KEY

Show work!

(32) 1. Let $z = 3x^2 + xy^2 + 2$.

a. Find the partial derivatives $\frac{\partial z}{\partial x}$, $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial x \partial y}$

b. Find the equation of the tangent plane at (1.1.6).

c. Find the directional derivative $\frac{dz}{du}$ where u is in the direction of $i-j$. at (1,1)

d. Find Δz_{tan} at (2,3) with $\Delta x = .01$ and $\Delta y = -.03$.

a. $\frac{\partial z}{\partial x} = 6x + y^2 \qquad \frac{\partial z}{\partial y} = 2xy$

$\frac{\partial^2 z}{\partial x^2} = 6 \qquad \frac{\partial^2 z}{\partial x \partial y} = 2y$

b. $\vec{N} = 7i + 2j - k$

$7(x-1) + 2(y-1) - (z-6) = 0$

$7x + 2y - z = 3$

c. $u = \frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j$

$\nabla z = (6x+y^2) i + 2xy\, j$

$= 7i + 2j$

$\frac{dz}{du} = 7i + 2j \cdot \frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j$

$= \frac{7}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

d. $\Delta z_{tan} = (12+9)(.01) + 2 \cdot 2 \cdot 3(-.03)$

$= 21(.01) + 12(-.03)$

$= .21 - .36 =$ [illegible]

$= -.15$

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(8) 2. Find ∇w for the function $w = x\sin y - y^2$.

$$\nabla w = \sin y\, \bar{i} + (x\cos y - 2y)\bar{j}$$

(10) 3. Approximate $\sqrt{(2.01)^3 + (.98)^2}$. $(2,1)$ $\Delta x = .01$ $\Delta y = -.02$

$$z = \sqrt{\cancel{x}\,x^3 + y^2} \qquad \frac{\partial z}{\partial x} = \frac{1}{2}(x^3+y^2)^{-1/2} 3x^2$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}(x^3+y^2)^{-1/2} 2y$$

$$\Delta z \approx \frac{3\cancel{4}}{2\sqrt{9}}(.01) + \frac{\cancel{2}(1)(.02)}{\cancel{2}\sqrt{9}} = \frac{2.01}{\cancel{2}} + \frac{(.02)}{3} = .\cancel{005 + .333}$$

$.006$

$.02\ \bar{4}\ .00\bar{6}$

$.014$

$$z \approx 3 + .005\,d.\,333 = 3.\cancel{338}$$

$.\cancel{014}$ 3.014 or $3.01\bar{3}$

(10) 4. What is the direction of maximum increase for the function $f(x,y,z) = x^2y + x^3y^3 - 6z$ at $(1,1,1)$?

$$\nabla f = (2xy + 3x^2y^3)\bar{i} + (x^2 + 3x^3y^2)\bar{j} - 6\bar{k}$$

$$= (2+3)\bar{i} + (1+3)\bar{j} - 6k$$

$$= 5\bar{i} + 4\bar{j} - 6\bar{k}$$

(10) 5. At what point(s) on the surface $x^3 + yz^2 = 0$ is the tangent plane parallel to the plane $6x + y - 3 = 0$?

$$w = x^3 + yz^2 \qquad w = 0$$

$$\nabla w = 3x^2\bar{i} + z^2\bar{j} + 2yz\bar{k}$$

$$\bar{N} = 6\bar{i} + \bar{j} + 0\bar{k}$$

$x = \pm\sqrt{2}\,z$ $\quad y = 0$

or $(0, 0, 0)$
y axis.

$$3x^2 = 6k \qquad z^2 = k \qquad 2yz = 0$$

$$x^2 = 2k \qquad z = \pm\sqrt{k} \qquad y = 0$$

$$x = \pm\sqrt{2k}$$

$(\sqrt{2}, 1, 0)$ $(\sqrt{2}, -1, 0)$

$(-\sqrt{2}, 1, 0)$ $(-\sqrt{2}, -1, 0)$

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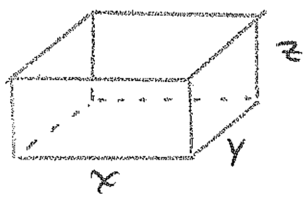
6. The point (0,0) is a critical point for the function f(x,y) $f(x,y) = x^3 - 6y^2 + 2$. Is this a maximum, minimum, or neither? Show!

can avoid issue!

$$\Delta f = (\Delta x)^3 - 6(\Delta y)^2 + 2 - 2$$

both

0) 7. A 2 cu. ft. box with no top is th be made with minimum material cost. The bottom material costs $1 per sq. ft. and the sides are made of material costing $2 per sq. ft. What are the dimensions of the box?



$$xyz = 2 \qquad z = \frac{2}{xy}$$

$$C = xy + 2[2xz + 2yz]$$

$$= xy + 4xz + 4yz$$

$$= xy + \cancel{8}y^{-1} + \cancel{8}x^{-1}$$

$$\frac{\partial C}{\partial x} = y - \cancel{8}x^{-2} = 0 \qquad \frac{\partial C}{\partial y} = x - \cancel{8}y^{-2} = 0$$

$$x^2y - \cancel{8} = 0 \qquad xy^2 - \cancel{8} = 0$$

$$y = \cancel{8}/x^2 \qquad x\,\frac{64}{x^4} = \cancel{8}$$

$$\cancel{16}\,\frac{8}{x^3} = 1$$

$$x = 2^{4/3}$$

$$y = \frac{4}{2^{8/3}}\,2$$

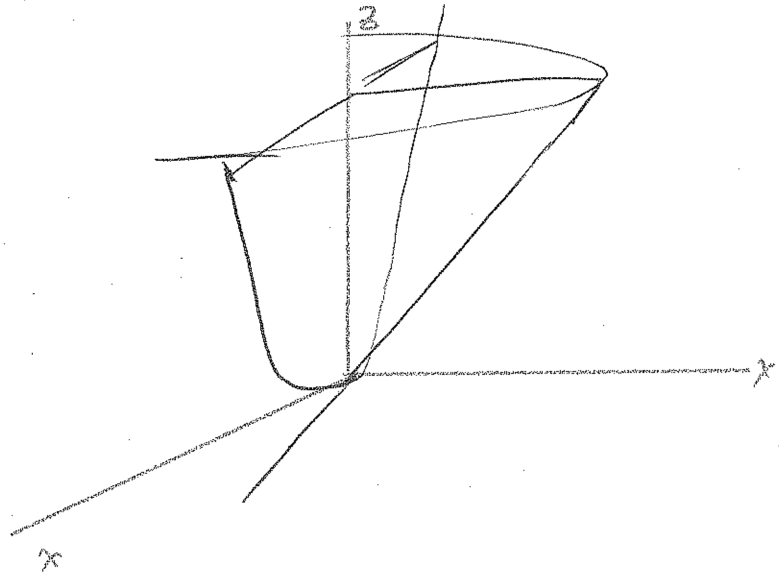
$$= \cancel{2^{4/3}}$$

$$z = \frac{2}{\cancel{2^{8/3}}}$$

$$= \frac{1}{2}\cancel{^{-4/3}}$$

$$2 \times 2 \times \frac{1}{2}$$

(o) 8. Sketch: $z = x^2 + y$



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