Math 233
Test II
May 19, 1978

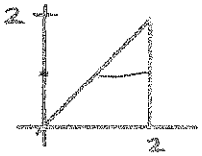
Name KEY

33
50
17
with intpts
maybe long

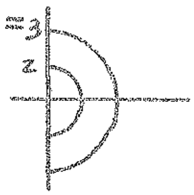
(30) 1. Compute:

a. $\int_1^2 \int_1^{\sqrt{x}} x+y \; dy \; dx$

b. $\iint_A x^2 y \; dx \; dy$ where A =



c. $\iint_B \sqrt{x^2+y^2} \; dx \; dy$ where B =



a. $\int_1^2 xy + \frac{y^2}{2}\Big|_1^{\sqrt{x}} dx = \int_1^2 x^{3/2} + \frac{x}{2} - x - \frac{1}{2} - \frac{x}{2} \; dx = \frac{x^{5/2}}{5/2} - \frac{x^2}{4} - \frac{x}{2}\Big|_1^2$

$= \frac{8\sqrt{2}}{5} - 1 - 1 - \left[\frac{2}{5} - \frac{1}{4} - \frac{1}{2}\right]$

$= \frac{8\sqrt{2}}{5} - \frac{2}{5} - 1\frac{1}{4}$

$= \boxed{\frac{8\sqrt{2}}{5} - \frac{33}{20}}$

(α)

b. $\int_0^2 \int_y^2 x^2 y \, dx \, dy = \int_0^2 \frac{x^3 y}{3}\Big|_y^2 dy = \int_0^2 \frac{8y}{3} - \frac{y^4}{3} \, dy = \frac{8}{3}\frac{y^2}{2} - \frac{y^5}{15}\Big|_0^2$

$= \cancel{2 - \frac{32}{15}} = \frac{16}{3} - \frac{32}{15} = \frac{48}{15} = \frac{16}{5}$

c. $\int_{-\pi/2}^{\pi/2} \int_2^3 r \, r \, dr \, d\theta = \int_{-\pi/2}^{\pi/2} \frac{r^3}{3}\Big|_2^3 d\theta = \int_{-\pi/2}^{\pi/2} 9 - \frac{8}{3} \, d\theta = \pi\left(\frac{19}{3}\right)$

(1)

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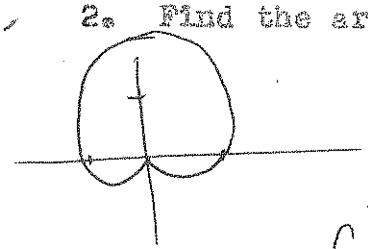
2. Find the area within the cardioid $r = 1 + \sin\theta$.



$$\int_0^{2\pi}\int_0^{1+\sin\theta} r\,dr\,d\theta = \int_0^{2\pi} \frac{r^2}{2}\Big|_0^{1+\sin\theta} d\theta = \int_0^{2\pi} \frac{(1+\sin\theta)^2}{2}\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} + \sin\theta + \frac{\sin^2\theta}{2}\,d\theta = \int \frac{3}{4}\theta - \cos\theta - \frac{\sin 2\theta}{8}\Big|_0^{2\pi}$$

$$\frac{1-\cos 2\theta}{4}$$

$$\frac{3}{4} + \sin\theta - \frac{\cos 2\theta}{4}$$

$$= \frac{6\pi}{4} - 1 + 0 - [0 - 1 + 0]$$

$$= \frac{3\pi}{2}$$

(21) 3. Find the sum of each series. If it does not converge, say so.

a. $\sum_{k=1}^{\infty} \frac{5}{3^k}$ b. $\sum_{k=2}^{\infty} \frac{5^k}{4^{k+1}}$ c. $\frac{3}{2^4} - \frac{3}{2^6} + \frac{3}{2^8} - \frac{3}{2^{10}} \pm \ldots$

a. $$= \frac{5}{3}\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^{k-1} = \frac{5}{3}\,\frac{1}{1-\frac{1}{3}} = \frac{5}{3}\,\frac{3}{2} = \frac{5}{2}$$

b. $$\frac{5^2}{4^3}\sum_{k=2}^{\infty}\frac{5^{k-2}}{4^{k-2}}$$ $$\left(\frac{5}{4}\right)^{k-2}$$ $r > 1$ divergs

c. $$\sum_{k=2}^{\infty}\frac{3(-1)^{k}}{2^{2k}} = \frac{3}{4}\sum_{k=2}^{\infty}\frac{(-1)^k}{4^k} = \frac{3(-1)^2}{4^2}\sum_{k=2}^{\infty}\left(\frac{-1}{4}\right)^{k-2} = \frac{3}{16}\,\frac{1}{1+1/4} = \frac{3}{16}\,\frac{4}{5} = \frac{3}{20}$$

$\frac{1}{2}$ 7

4. Write as a fraction: 1.2474747...

$$1 + \frac{2}{10} + \frac{47}{10^3} + \frac{47}{10^5} + \frac{47}{10^7} + \cdots$$

$$1 + \frac{2}{10} + \frac{1}{10^3}\sum_{k=0}^{\infty}\frac{47}{10^{2k}} = 1 + \frac{2}{10} + \frac{47}{10^3}\,\frac{1}{1-1/100}$$

$$= \frac{47}{10}\,\frac{100}{99}$$

$$= 1 + \frac{2}{10} + \frac{47}{990}$$

$$= \frac{1188}{990} + \frac{47}{990} = \frac{1235}{990}$$

12 · 99 = 108, 108 → 1188; 1188 + 47 = 1235

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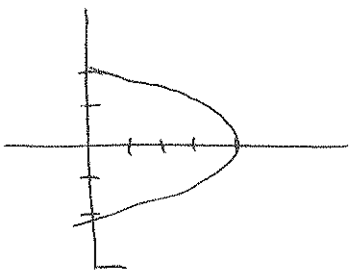
43 44

(15) 5. Find the center of mass (centroid) of the region bounded by the curve $x = 4 - y^2$ and the y axis.



$y = 0$

(3)

$$\int_{-2}^{2}\int_{0}^{4-y^2} dx\,dy = \int_{-2}^{2} 4-y^2\,dy = 2\left(4y - \frac{y^3}{3}\Big|_{0}^{2}\right) = 8$$

Hint

$$2\left(8 - \tfrac{8}{3}\right) = 16\left(\tfrac{2}{3}\right) = \boxed{\tfrac{32}{3}}$$

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$$\int_{-2}^{2}\int_{0}^{4-y^2} x\,dx\,dy$$

$$= \int_{-2}^{2} \frac{x^2}{2}\Big|_{0}^{4-y^2} dy = \int_{-2}^{2} \frac{(4-y^2)^2}{2}\,dy = \int_{-2}^{2} \frac{16-8y+y^2}{2}\,dy$$

$8 - 4y^2 + \frac{y^4}{2}$

$= 8y - \ldots = 16 - 8 - \frac{8}{6} - \left(-16 - 8 + \frac{8}{6}\right)$

$32 - \frac{8}{3}$

$16 - 8 + \frac{8}{6} - \left(-16 - 8 - \frac{8}{6}\right)$

$32 + \frac{8}{3} = \frac{104}{3}$

$\bar{x} = \frac{104/3}{32/3} = \frac{13}{4}$

$\left(\frac{13}{4}, 0\right)$

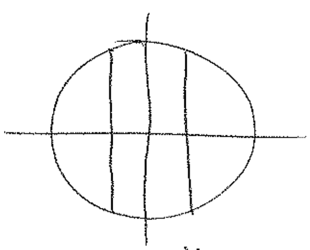
(6)

(12) 6. A region is bounded by the sphere centered at (0,0,0) with radius 4 and lies within the cylinder $x^2+y^2 = 9$. $x^2+y^2+z^2=16$

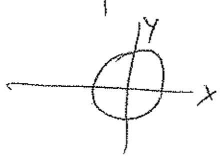
Set up volume as an integral in

i. cartesian coordinates.

ii. cylindrical coordinates.



a. $$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{-\sqrt{16-x^2-y^2}}^{\sqrt{16-x^2-y^2}} dz\,dy\,dx$$



b. $$\int_{0}^{2\pi}\int_{0}^{3}\int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta$$

$$\int_{-2}^{2} \frac{16-8y^2+y^4}{2}$$

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$$= \int_{-2}^{2} 8 - 4y^2 + \frac{y^4}{2}\,dy = 8y - \frac{4y^3}{3} + \frac{y^5}{10}\Big|_{-2}^{2}$$

$$= 16 - \frac{32}{3} + \frac{32}{10} - \left(-16 + \frac{32}{3} - \frac{32}{10}\right)$$

$$= 32\left[1 - \frac{2}{3} + \frac{1}{5}\right]$$

$$32 \cdot \frac{15-10+3}{15} = 32\left(\frac{8}{15}\right) = \frac{256}{15}$$

$$\frac{256/15}{32/3} = \frac{8}{5}$$

$$\left(\frac{8}{5}, 0\right)$$

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