

$\mathcal{B} = 23, 3$   
 $m = 23$

attend 25 min  
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 all by 25

Name Key

1. Definitions:

a. A subset  $H$  of a vector space  $V$  is called a subspace of  $V$  if and only if

16/26

b. The vectors  $b_1, b_2, \dots, b_p$  form a basis for a vector space  $V$  if and only if

18/26  
 about  
 all

2. Find a basis for the subspace spanned by the vectors  $(1, 1, -2, 3), (-3, -3, 8, -6), (0, 0, 4, 6)$ .  
 This is a 2 dimensional subspace of  $\mathbb{R}^4$ .

13

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ -3 & -3 & 8 & -6 \\ 0 & 0 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(1, 1, -2, 3) \quad (0, 0, 2, 3)$$

3. The matrix  $A = \begin{bmatrix} 1 & 2 & 2 & 0 & 1 \\ -4 & -7 & -6 & -3 & -4 \\ 3 & 8 & 10 & -6 & 3 \end{bmatrix}$  is row equivalent to  $\begin{bmatrix} 1 & 2 & 2 & 0 & 1 \\ 0 & 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

a. Find a basis for the row space of  $A$ :

9/11

$$(1, 2, 2, 0, 1) \quad (0, 1, 2, -3, 0)$$

b. Find a basis for the column space of  $A$ :

$$(1, 4, 3) \quad (2, -7, 8)$$

c. The row space of  $A$  is a 2 dimensional subspace of  $\mathbb{R}^5$ .

d. The column space of  $A$  is a 2 dimensional subspace of  $\mathbb{R}^3$ .

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e. The rank of  $A$  is 2.

f. The nullspace of  $A$  is a 3 dimensional subspace of  $\mathbb{R}^5$ .

g. Find a basis for the null space of  $A$ :

$$\begin{aligned} x_5 = t \quad x_4 = s, \quad x_3 = u \\ x_2 + 2u - 3s = 0 \\ x_2 = -2u + 3s \\ x_1 + 2(-3u + 3s) + 2u + t = 0 \\ x_1 = 2u - 6s + t \end{aligned}$$

$$t \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$(2, -2, 1, 0, 0) \\ (-6, 3, 0, 1, 0) \\ (1, 0, 0, 0, 1)$$

4. Find a basis for the subspace of  $M_{22}$  (the  $2 \times 2$  matrices) consisting of matrices where the 1,1 entry is 0. What is the dimension of this subspace?

4

$$\begin{bmatrix} 0 & b \\ c & d \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad 3$$