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MATH 332
Quiz III
October 10, 1990 Name Key

$\bar{x}$ 24.4/30

m = 26

(5) 1. Complete and prove:
If the matrix A is invertible, then det A $\neq 0$.

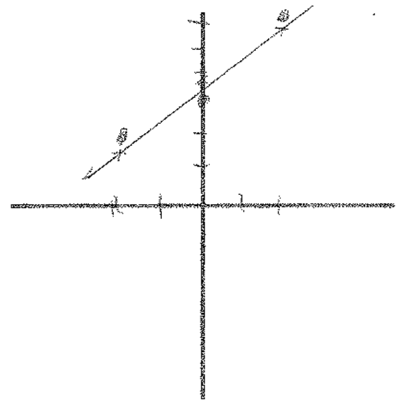
16/24

$AA^{-1} = I$

$\det A \det A^{-1} = \det I = 1$

so $\det A \neq 0$.

(5) 2. Find the equation of the least squares line for the data:
(-2,2), (0,3), (2,6). Plot the points and the line.



$M = \begin{bmatrix} -2 & 1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$ $M^tM = \begin{bmatrix} -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 3 \end{bmatrix}$

$M^ty = \begin{bmatrix} -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$

$M^tM \begin{bmatrix} m \\ b \end{bmatrix} = M^t y$

$\begin{bmatrix} 8 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$ $m = 1$, $b = \frac{11}{3}$

9/24

$\boxed{y = x + \frac{11}{3}}$

-6

x	y
-2	5/3
0	11/3
2	17/3

(4) 3. Find the quadratic polynomial which best fits the following points.
(2,3), (3,5), (4,6), (6,5)
[Write the equations in matrix form. Do NOT solve.]

next time

14/24

$M = \begin{bmatrix} 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \\ 36 & 6 & 1 \end{bmatrix}$ $y = ax^2 + bx + c$ $\bar{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $y = \begin{bmatrix} 3 \\ 5 \\ 6 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 4 & 9 & 16 & 36 \\ 2 & 3 & 4 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \\ 36 & 6 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 & 9 & 16 & 36 \\ 2 & 3 & 4 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 6 \\ 5 \end{bmatrix}$

14 4. Find the determinant of the following matrices: (Use any valid method. Show work. Think!)

2/2

a. $\det\begin{bmatrix} 2 & 8 \\ 9 & -2 \end{bmatrix} = -4 - 72 = -76$

12/

b. $\det\begin{bmatrix} 1 & 8 & 0 & 1 \\ 9 & 3 & -3 & 0 \\ 2 & 0 & 4 & 2 \\ 1 & 0 & -1 & -2 \end{bmatrix} = -8\begin{vmatrix} 9 & -3 & 0 \\ 2 & 4 & 2 \\ 1 & -1 & -2 \end{vmatrix} + 3\begin{vmatrix} 1 & 0 & 1 \\ 2 & 4 & 2 \\ 1 & -1 & -2 \end{vmatrix}$

$-8\left[9\begin{vmatrix} 4 & 2 \\ -1 & -2 \end{vmatrix} - (-3)\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix}\right] + 3\left[\begin{vmatrix} 4 & 2 \\ -1 & -2 \end{vmatrix} + \begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix}\right]$

c. $\det\begin{bmatrix} 1 & 7 & 9 \\ 0 & 2 & 9 \\ 1 & 7 & 9 \end{bmatrix}$ 0

$-8[9(-6) + 3(-6)] + 3[-6 - 6]$

$-8(-90) + 36 =$ ~~432+12=444~~ $\boxed{540}$

54
8

32

720
-54
-18

-72
8

576

d. $\det\begin{bmatrix} 0 & 2 & 5 & 6 & 10 \\ 0 & 0 & 4 & 3 & 9 \\ 0 & 0 & 0 & -3 & 6 \\ 0 & 0 & 0 & 0 & 1 \\ 2 & 2 & 3 & 4 & 5 \end{bmatrix} = 2\begin{vmatrix} 2 & 5 & 6 & 10 \\ 0 & 4 & 3 & 9 \\ 0 & 0 & -3 & 6 \\ 0 & 0 & 0 & 1 \end{vmatrix}$

$= 2\,|-24| = -48$