

1. Show that $(1, 5, -4)$ is in the span of $(2, 4, -2)$ and $(2, 3, -1)$ by writing it as a linear combination. Set up equations and solve. *OR NOT!*

$$a_1(2, 4, -2) + a_2(2, 3, -1) = (1, 5, -4)$$

$$\left[\begin{array}{cc|c} 2 & 2 & 1 \\ 4 & 3 & 5 \\ -2 & -1 & -4 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 2 & 2 & 1 \\ 0 & -1 & 3 \\ 0 & 1 & -3 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 2 & 2 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$a_2 = -3$$

$$2a_1 + 6 = 1 \quad 2a_1 = -5 \quad a_1 = -\frac{5}{2}$$

$$\frac{7}{2}(2, 4, -2) - 3(2, 3, -1)$$

2. The vectors $(2, 0, 4)$ and $(1, 3, 2)$ do not span \mathbb{R}^3 . Find the algebraic condition that the vector (x_1, x_2, x_3) must satisfy to be in the span of these vectors. (This will be the equation of the plane through $(0, 0, 0)$ containing these vectors.)

$$\left[\begin{array}{cc|c} 2 & 1 & x_1 \\ 0 & 3 & x_2 \\ 4 & 2 & x_3 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 2 & 1 & x_1 \\ 0 & 3 & x_2 \\ 0 & 0 & x_3 - 2x_1 \end{array} \right] \quad y?$$

$$x_3 - 2x_1 = 0 \quad x_3 = 2x_1$$

(Some said $x_3 = 3$)

3. Verify that the set of vectors (x_1, x_2, x_3) for which $x_1 + x_2 + x_3 = 0$ is a subspace of \mathbb{R}^3 .

a) $0 + 0 + 0 = 0$ ✓

c) $x_1 + x_2 + x_3 = 0 \quad y_1 + y_2 + y_3 = 0$

$$(x_1 + y_1, x_2 + y_2, x_3 + y_3) \rightarrow x_1 + y_1 + x_2 + y_2 + x_3 + y_3$$

$$= x_1 + x_2 + x_3 + y_1 + y_2 + y_3$$

$$= 0 + 0 = 0 \quad \checkmark$$

4. Show that the set of vectors (x_1, x_2) for which $x_1 = 0$ or $x_2 = 0$ is NOT a subspace of \mathbb{R}^2 .

$(0, 0)$ ✓

$(1, 0) + (0, 1) = (1, 1)$ not in set.

5. Find the quadratic polynomial which passes through the points $(-1, 2)$, $(0, 3)$, and $(1, 7)$. Set up equations, and solve.

$$y = a + bt + ct^2 \quad 2 = a - b + c$$

$$3 = a$$

$$7 = a + b + c$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 3 \\ 1 & 1 & 1 & 7 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & 0 & 5 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -2 & 3 \end{array} \right]$$

$$c = \frac{3}{2} \quad b - \frac{3}{2} = 1 \quad b = \frac{5}{2}$$

$$a - \frac{5}{2} + \frac{3}{2} = 2 \quad a = 3$$

$$y = 3 + \frac{5}{2}t + \frac{3}{2}t^2$$

2/3