

1. Prove:

If C and B are inverses of the matrix A, then B = C.

$$AC = I$$

$$BAC = B \cdot I = B$$

$$(BA)C = B$$

$$IC = B$$

$$C = B$$

(2) 2. Find the inverse of the following matrix (Think!):

$$\text{inv} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ self.}$$

(3) 3. Give the elementary matrix which performs the row operation of adding 3 times the third row to the second row of a 3x3 matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

(4) 4. In matrix algebra, can we say  $(A+B)^2 = A^2 + 2AB + B^2$ ? Either justify, or else what can we say? *NO*

$$(A+B)^2 = A^2 + AB + BA + A^2$$

(5) 5. Find the determinant of the following matrices: (Show work. Think!)

$$\begin{bmatrix} 2 & 9 \\ 8 & 2 \end{bmatrix} = 4 - 72 = -68$$

$$\begin{bmatrix} 2 & 0 & 8 & -6 & 4 \\ 2 & 0 & 8 & -6 & 4 \\ 9 & 8 & 5 & -8 & 2 \\ 0 & 6 & -7 & 8 & 2 \\ 0 & 4 & 3 & -1 & 1 \end{bmatrix} \Rightarrow \text{2 rows same.}$$

$$\begin{bmatrix} 2 & 8 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} = 2 \cdot 2 \cdot (-1) = -4 \quad (\Delta)$$

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ -1 & 2 & -1 & 0 \\ 2 & 2 & 10 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix} = -2 \det \begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & -1 \\ 1 & 2 & 1 \end{bmatrix} = -2 \left[ \begin{array}{l} |2 \ -1| \ -2 | -1 \ -1| \\ |2 \ 1| \end{array} \right] + 4 \begin{array}{l} | -1 \ 2| \\ | 1 \ 2| \end{array}$$

$$= -2 \left[ 4 + 0 + 4(-4) \right]$$

$$= -2(-12) = 24$$