MATH 332
Quiz IV
March 16, 1988 Name Answers

Briefly justify your answers.

(5) 1. Complete the following definition: The vectors $\bar{v}_1, \bar{v}_2, \cdots, \bar{v}_n$ are linearly independent if and only if

2. Is the vector (8,1,13,12) in the space spanned by the vectors (1,-1,2,0) and (2,1,3,4)?

(6)

$a(1,-1,2,0) + b(2,1,3,4) \overset{?}{=} (8,1,13,12)$

$a + 2b = 8$
$-a + b = 1$
~~xxxxxx~~
$2a + 3b = 13$
$4b = 12$

$b = 3$
$-a + b = 1 \quad a = 2$
$2 + 2(3) = 8$ ✓
$2(2) + 3(3) = 13$ ✓

or
$\left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$
has solution.

Yes.

3. Verify that the set of matrices of the following form is a vector space. What is the dimension of this space? (a,b are any real numbers.)

(7)

$\begin{bmatrix} a & a \\ b & b \end{bmatrix}$

Since a subset of M_{22} only need to show closure.

(1) $\begin{bmatrix} a & a \\ b & b \end{bmatrix} + \begin{bmatrix} c & c \\ d & d \end{bmatrix} = \begin{bmatrix} a+c & a+c \\ b+d & b+d \end{bmatrix}$ of same form;

(2) $k\begin{bmatrix} a & a \\ b & b \end{bmatrix} = \begin{bmatrix} ka & ka \\ kb & kb \end{bmatrix}$ also same form.

Hence a subspace of M_{22}.

dimension = 2 Basis $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$.

(7) 4. Find a basis for the solution space of the following system of equations.

$x_1 + 2x_2 + x_3 - x_4 = 0$
$x_2 - x_3 + x_4 = 0$
$-x_2 + x_3 - x_4 = 0$

This is a subspace of $R^{\boxed{}}$ 4. (Fill in.)

$\begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_4 = t$
$x_3 = s$
$x_2 - s + t = 0$
$x_2 = s - t$
$x_1 + 2(s-t) + s - t = 0$

$x_1 = -3s + 3t$
$(x_1, x_2, x_3, x_4) = s(-3,1,1,0) + t(3,-1,0,1)$

Basis:
$(-3,1,1,0), (3,-1,0,1)$