

MATH 332

Quiz IV

October 19, 1980

Name KEY

- (W) 1. Let $u = (1, -2, 4, 1)$, $v = (-1, 2, 1, 0)$, $w = (2, -3, 1, 0)$. Find (if possible):

a. $u + 4v = (1, -2, 4, 1) + (-4, 8, 4, 0) = \underline{(-3, 6, 8, 1)}$

b. $u \cdot w = (1, -2, 4, 1) \cdot (2, -3, 1, 0) = 2 + 6 + 4 + 0 = \underline{(12)}$

c. $\|u\| = \sqrt{1^2 + (-2)^2 + 4^2 + 1^2} = \sqrt{22}$

d. Find a unit vector parallel to v . $\frac{1}{\|v\|} v = \frac{1}{\sqrt{6}} (-1, 2, 1, 0)$

e. $u \cdot (v \cdot w) = \text{not possible}$

2. Find the (orthogonal) projection of the vector $u = (-2, 2, 0)$ onto the vector $v = (2, -1, 2)$.

$$\begin{aligned} \frac{u \cdot v}{\|v\|} \cdot v &= \frac{(-2, 2, 0) \cdot (2, -1, 2)}{(2, -1, 2) \cdot (2, -1, 2)} (2, -1, 2) = \frac{-6}{9} (2, -1, 2) \\ &= -\frac{2}{3} (2, -1, 2) \\ &= \left(-\frac{4}{3}, \frac{2}{3}, -\frac{4}{3} \right) \end{aligned}$$

3. Give the equation of the plane in \mathbb{R}^3 which contains $(1, 2, 0)$ and is parallel to the plane $x + 2y - 3z = 3$.

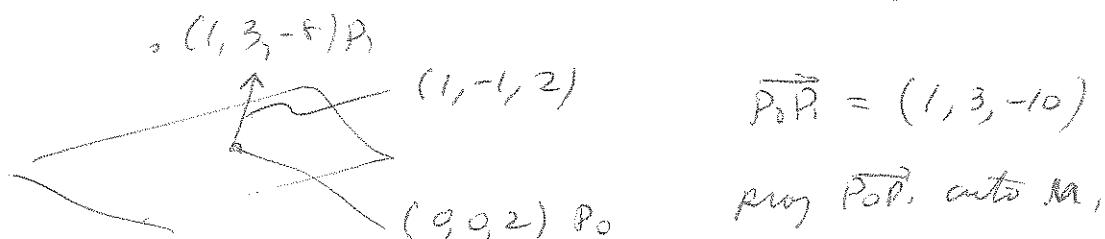
$$\begin{aligned} P_0 &= (1, 2, 0) & (1, 2, -3) \cdot (x-1, y-2, z) = 0 & \text{point} \\ \vec{n} &= (1, 2, -3) & (x-1) + 2(y-2) - 3z = 0 & \text{point} \\ & & x + 2y - 3z - 5 = 0 & \text{to guess} \\ & & \text{or } x + 2y - 3z = 5 & \text{fraction} \end{aligned}$$

4. Find the equation(s) of the straight line through the point $(2, 0, -6)$ which is perpendicular to the plane $x + 3y - z = 9$.

$$\begin{aligned} \vec{m} &= t(1, 3, -1) \\ \vec{m} &= (1, 3, -1) \quad \vec{b} = (2, 0, -6) \\ \vec{v} &= \vec{m}t + \vec{b} = t(1, 3, -1) + (2, 0, -6) \\ &= (t+2, 3t, -t-6) \end{aligned}$$

or $x = t+2, y = 3t, z = -t-6$

5. Find the distance from the point $(1, 3, -8)$ to the plane $x - y + 2z = 4$



$$\begin{aligned} \vec{P} &= \frac{(1, 3, -8) - (9, 0, 2)}{(1, -1, 2) \cdot (1, -1, 2)} (1, -1, 2) = \frac{1 - 3 - 20}{6} (1, -1, 2) \\ &= -\frac{22}{6} (1, -1, 2) \end{aligned}$$

$$\|\vec{P}\| = \sqrt{\frac{22^2}{6}} = \frac{22}{\sqrt{6}} = \frac{22\sqrt{6}}{6} = \frac{11}{3}\sqrt{6}$$