

(5)

1. Complete the Definition: The set of vectors v_1, v_2, \dots, v_n is said to be linearly independent if and only if

the only way to $a_1 v_1 + \dots + a_n v_n = 0$
 $\Rightarrow a_1 = 0, a_2 = 0, \dots, a_n = 0,$

8

(12)

2.

$v_1 = \begin{bmatrix} 2 \\ -8 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ -7 \\ 5 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 3 \\ 11 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 3 \\ 9 \end{bmatrix}$

a. Are vectors v_1, v_2, v_3 linearly independent?

$a_1 \begin{bmatrix} 2 & 2 & 0 \\ -8 & -7 & 3 \\ 2 & 5 & 11 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

14

yes

b. Are vectors v_1, v_2, v_4 linearly independent?

15

$\begin{bmatrix} 2 & 2 & 0 \\ -8 & -7 & 3 \\ 2 & 5 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

no

(5)

3. Is the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ in the null space of the matrix $\begin{bmatrix} 2 & 3 \\ -2 & 5 \end{bmatrix}$?

$\begin{bmatrix} 2 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 16 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ NO

13

↓ after 5 min

(8)

4. a. Find a basis for the subspace spanned by the vectors $(2, 2, 0), (-8, -7, 3)$ and $(2, 5, 9)$.

b. This is a subspace of \mathbb{R}^3 ? (R something!)

13

(A) $\begin{bmatrix} 2 & 2 & 0 \\ -8 & -7 & 3 \\ 2 & 5 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$(2, 2, 0)$ and $(0, 1, 3)$

(B) $\begin{bmatrix} 2 & -8 & 2 \\ 2 & -7 & 5 \\ 0 & 3 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -8 & 2 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -8 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$a_3 = 1, a_2 = -3, a_1 + 24 + 2 = 0$
 $a_1 = -26$

$(2, 2, 0), (-8, -7, 3)$